

## Exploratory stress-scenario analysis with applications to EMU

Peter Zangari  
Morgan Guaranty Trust Company  
Risk Management Research  
(1-212) 648-8641  
zangari\_peter@jpmorgan.com

It is difficult to argue with the notion that history is rather limited in terms of predicting the future, especially when events such as EMU have the potential to render history useless.<sup>1</sup> Therefore, to effectively measure financial risk in times of political or economic events such as EMU, risk managers should (1) consider alternative sources of information, such as option prices, to predict changes in market prices, and/or (2) develop a tool to ‘play out’ how potentially large market moves<sup>2</sup> affect the risk of their portfolio. The implementation of the first alternative is hampered by the lack of reliable prices on options across a wide range of asset classes. As for the second alternative, risk managers typically rely on stress-tests to quantify the effect that large market moves have on their portfolio’s value. Unfortunately, as shown by Kupiec (1998), the standard application of stress-tests is inadequate and, therefore, rather limited in terms of the information it can offer.<sup>3</sup>

The focus of this article is on providing a methodology to implement the second alternative. We propose a tool to measure Value-at-Risk<sup>4</sup> (VaR) associated economic and financial events, and demonstrate how it can be used to measure the financial risk associated with **EMU-related stress scenarios**. This tool, which we refer to as *ESSA*—Exploratory Stress Scenario Analysis—allows risk managers to automatically and consistently adjust VaR estimates to reflect their views on market price movements, market uncertainty and co-movements among markets.

**ESSA allows users to quantify their beliefs about future market scenarios by (1) offering a mechanism to translate these beliefs into statistical parameter estimates and (2) combine these parameter estimates with parameter estimates based on historical prices.** The relationship between market scenarios and the statistical parameters that enter the VaR calculation can be described by the following three examples:

- *A large drop in market prices can be reflected in a relatively large, negative mean forecast.* History has shown that the level of some financial prices can change rather dramatically over relatively short periods of time. We can account for such price moves by adjusting expected returns on the appropriate market prices. For example, recently, certain Asian currencies as measured against the US dollar have demonstrated a substantial drop in value.
- *Increased market uncertainty can be described by increasing in price volatility.* It is not uncommon for risk managers to think of increased volatilities as representing a heightened state of market uncertainty.
- *Large, positive co-movements among market prices can be accounted for by increasing historical correlations.* Depending on the market environment, correlation estimates may change dramatically, that is, either decrease (break down) or increase (lock up) in absolute value. An example of the latter involves the recent events in the dollar price of Asian currencies.<sup>5</sup>

<sup>1</sup> Note that this problem is not specific to EMU. It is well known that structural regimes, or ‘breaks’, can severely complicate forecasting procedures that are based on time series.

<sup>2</sup> Such ‘market moves’ would either be too large to be captured by a typical Value-at-Risk system or simply be completely inconsistent with historical data. This inconsistency may result from the fact that there is a relatively large probability of an impending event.

<sup>3</sup> Kupiec (1998) demonstrates the problem with standard stress-tests and proposes a new methodology to carry out such tests.

<sup>4</sup> We assume that the variance/covariance methodology is used to measure VaR. However, as shown below, our methodology applies to VaR calculations based on Full Monte Carlo Simulation as well as delta/gamma.

<sup>5</sup> See Finger, “A methodology to stress correlations,” RiskMetrics Monitor, Fourth Quarter, 1997.

## Exploratory stress-scenario analysis with applications to EMU (continued)

*ESSA* allows you to visualize how your risk profile, measured in terms of VaR, changes in response to changes in parameters that determine VaR estimates—namely, means, volatilities and correlations of market prices. Moreover, it allows you to calculate Value-at-Risk over a range of weights that are given to a particular scenario. The result is that you can see how your risk profile changes as the relative importance of a particular scenario changes.

As will become more clear below, *ESSA* is a considerable enhancement to standard stress-testing for three reasons. First, *ESSA* casts stress-testing in a standard VaR setting so that comparisons between typical VaR estimates and “stress-test” risk estimates are consistent. Second, *ESSA* allows users to quantify how the risk profile of a particular portfolio changes in response to (1) different stress scenarios and (2) the importance of these stress scenarios. And third, *ESSA* allows users to measure **systemic risk**, defined as the risk from infrequent events that are highly correlated across a large number of assets. Usually, stress-testing procedures focus exclusively on **catastrophic risk** where changes in correlation are ignored.

The rest of this article is organized as follows:

- In Section 2, we present the basic model used to generate “Scenario-VaR” estimates and provide two examples to demonstrate how *ESSA* works.
- In Section 3, we apply *ESSA* to a portfolio consisting of fixed income and equity positions to study how that portfolio’s value changes in response to five hypothetical EMU-related stress scenarios.
- In Section 4, we explain briefly how it is straightforward to apply *ESSA* to measure the market risk on option positions. In particular, we discuss how to implement *ESSA* when using Monte Carlo Simulation and delta/gamma methodologies.
- Section 5 summarizes the article’s main points and underscores the importance that changes in the mean have on VaR estimates as compared to volatilities and correlations.

### 1. The basic model and examples

*ESSA* relies on a model for market returns that is really quite simple and just an adaptation of the original RiskMetrics model, which applies the variance/covariance (VCV) or structured Monte Carlo Simulation VaR methodologies. All risk calculations are based on the assumption that market returns<sup>6</sup> can be described reasonably well by the multivariate normal distribution (MVN),<sup>7</sup> which implies that the parameters that determine VaR are means, variances and covariances. Mathematically, we can write this assumption as:

$$[1] \quad R_t \sim MVN(\mu, \Sigma)$$

where  $R_t$  represents an  $N \times 1$  vector of market returns at time  $t$  with mean vector  $\mu$  and covariance matrix  $\Sigma$ .

<sup>6</sup> We assume that returns on market factors (e.g., USD/DEM exchange rate) are conditionally normal. We do not, however, make any particular assumption about the distribution of returns on positions.

<sup>7</sup> This assumption may not be as unrealistic as it appears. Kupiec (1998) provides some evidence that the *conditional* multivariate normal distribution performs reasonably well at measuring risk associated with stress scenarios.

**Exploratory stress-scenario analysis with applications to EMU (continued)**

Our point of departure from the standard RiskMetrics model is the assumption that estimates of the statistical parameters that determine VaR consist of two parts: (1) an objective part, which is based on time series of market prices, such as those provided by RiskMetrics, and (2) a subjective part, which is based on one’s view on future market moves. As we show below, market views about a particular scenario enter the VaR calculation through the subjective mean and the covariance matrix.

To be more specific, we assume that the mean and the covariance matrix that enter the VaR calculation can be written as the weighted average of their respective data-based (objective) and market view (subjective) counterparts. Mathematically, the VaR estimate is based on a mean,  $\mu$ , and covariance matrix,  $\Sigma$ , given by:<sup>8</sup>

$$\begin{aligned}
 [2] \quad \mu &= \omega_{\mu}\mu_0 + (1 - \omega_{\mu})\hat{\mu} \\
 \Sigma &= \omega_{\Sigma}\Sigma_0 + (1 - \omega_{\Sigma})\hat{\Sigma}
 \end{aligned}$$

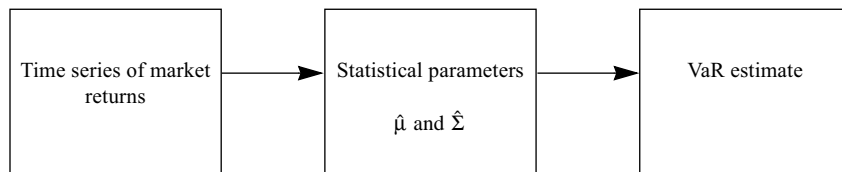
where

- $\hat{\mu}$  and  $\hat{\Sigma}$  are estimates for the mean and the covariance matrix, respectively, based on historical market prices (e.g., RiskMetrics). Chart 1 illustrates the relationship between time series of market returns, mean and covariance parameters, and the standard VaR (RiskMetrics) calculation.

*Chart 1*

**Standard VaR calculation**

*Objective parameter estimates (based on historical prices)*



- $\mu_0$  and  $\Sigma_0$  are the subjective or prior values for the mean and covariance matrix, respectively. That is,  $\mu_0$  and  $\Sigma_0$  represent the mean and covariance parameters associated with a specific market view on a scenario. The values of these parameters may simply be the result of a risk manager’s ex-ante view of future market moves or based on an alternative (data-based) estimate of the mean and covariance matrix.<sup>9</sup> Chart 2 (page 33) illustrates the relationship between a risk manager’s market view, statistical parameters and the VaR estimate.

<sup>8</sup> To avoid messy notation, throughout this article we ignore the time  $t$  designation. Readers should assume that all calculations are for time  $t$  using market information (prices) up to and including time  $t - 1$ .

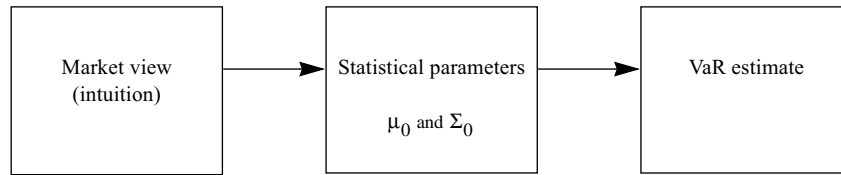
<sup>9</sup> For example, Kupiec (1998) explains how to appropriately adjust the mean and covariance matrix of market returns when conditioning on specific scenarios.

Exploratory stress-scenario analysis with applications to EMU (continued)

Chart 2

**VaR calculation based on market view**

*Subjective parameter estimates translate market opinion into statistical parameters*

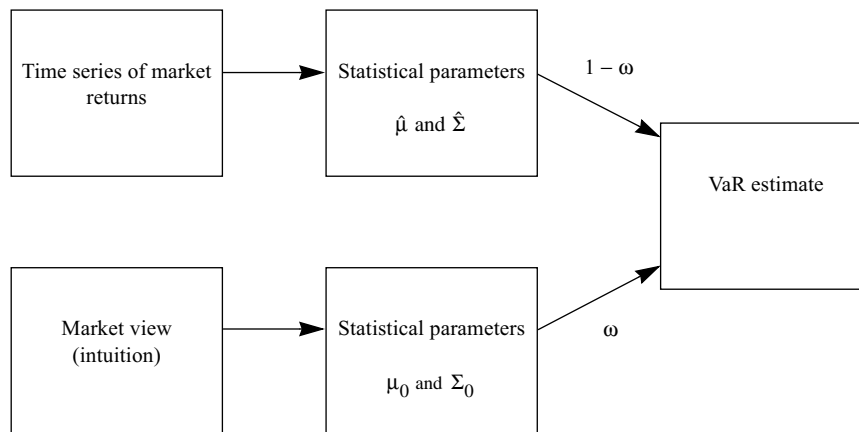


- $\omega_\mu$  and  $\omega_\Sigma$  are weights that take values from 0 to 1 (or 0% to 100%) and that are applied to the prior mean and covariance matrix, respectively. These weights determine the relative importance that is given to the prior mean and covariance matrix, i.e., the importance that should be given to a particular scenario. Throughout this article we assume that  $\omega = \omega_\mu = \omega_\Sigma$ ; however, nothing prevents us from relaxing this assumption. Chart 3 shows how these weights allow us to combine information from historical prices with a market view to yield one VaR estimate that reflects both types of information.

Chart 3

**VaR calculation based on combination of data and view**

*Update or replace historical information with your view on the market*



Calculating VaR over different values of  $\omega$  allows us to study how sensitive a portfolio's value is to a particular scenario.

ESSA's principal contribution to measuring market risk is that it offers a straightforward and consistent methodology for translating market views into statistical parameter estimates that go into the VaR calculation. Table 1 (page 34) presents the three parts to this translation process. Part 1 involves specifying a market view (e.g., the lira (ITL) drops 15% against the deutschemark (DEM) over the next day); part 2 involves converting this view into statistical parameter estimates (e.g., the one-day expected return (mean) on the ITL/DEM exchange rate increases 15%) and part 3 involves the methodology that allows views to

**Exploratory stress-scenario analysis with applications to EMU (continued)**

be quantified, i.e., the exact method of how market views are translated into changes in means, volatilities and correlations.

*Table 1*  
**Market view translation process**  
*Some typical market views*

	<b>Part 1</b> <i>Market view</i>	<b>Part 2</b> <i>Statistical parameters</i>	<b>Part 3</b> <i>Methodology</i>
1.	Price/rate changes by $x$ percent.	mean ( $\mu_0$ )	$\mu_0 = \hat{\mu} + x/100$
2.	Increase in market uncertainty by $x$ percent of historical value.	volatility ( $\sigma_0$ )	$\sigma_0 = \hat{\sigma}(1 + x/100)$
3.	Large moves among several prices are highly correlated (systemic risk).	correlation ( $\rho_0$ )	Increase historical correlations using methodology proposed by Finger (1997).
4.	Large moves among several prices are uncorrelated (catastrophic risk).	correlation ( $\rho_0$ )	Decrease historical correlations using methodology proposed by Finger (1997).

Table 1 shows how to translate four market views into changes in the parameters that govern VaR. Note that we did not choose means, volatilities and correlations as the parameters that are affected by market views simply because it is easy to do so. On the contrary, we made a deliberate choice to convert market views into changes in these parameters because market professionals tend to think (or at least speak) in terms of these parameters. Surely, it is our experience in working with clients, that risk managers and related industry professionals speak in terms of “correlation breakdowns”, “market contagion”, and “volatility shocks”. An important goal in producing *ESSA* is, therefore, to help risk managers get a better understanding of their market intuition and how it translates into VaR estimates.

As a final point in this section, note that the model we employ (Eq. [1]) is a simplified version of a more elaborate Bayesian statistical model, which we present in Appendix A. An alternative model that allows us to incorporate market views into the VaR calculation<sup>10</sup> assumes that market returns are generated from a mixture of two MVN distributions; the first with mean  $\hat{\mu}$  and covariance matrix  $\hat{\Sigma}$  and the second with mean  $\mu_0$  and covariance matrix  $\Sigma_0$ . We can write this model for market returns as

$$[3] \quad R_t \sim p \times MVN(\hat{\mu}, \hat{\Sigma}) + (1 - p) \times MVN(\mu_0, \Sigma_0)$$

where the parameter  $p$  ( $0 < p < 1$ ) denotes the probability that returns are sampled from the multivariate normal distribution with mean  $\hat{\mu}$  and covariance  $\hat{\Sigma}$ . From a practical perspective, the problem with the model described by Eq. [3] is that returns do not follow a multivariate normal distribution. In fact, the probability distribution of their weighted sum (i.e., the portfolio) does not have an easily computable expression. Therefore, the only practical way to generate VaR estimates from Eq. [3] would require Monte Carlo simulation where we use Eq. [3] to generate draws of  $R_t$  and then value our portfolio based on these draws. For large portfolios, this approach would be at best time consuming and at worst impractical. The actual improvement of using the model in Eq. [3] over the model in Eq. [1] has yet to be determined and is left for further study.

<sup>10</sup> See Zangari (1996) for an application of such a model.

## Exploratory stress-scenario analysis with applications to EMU (continued)

**1.1 An illustrative example**

To demonstrate how *ESSA* works, consider a simple hypothetical portfolio consisting of two U.S. stock positions as reported in Table 2.

Table 2

**Statistics for hypothetical stock portfolio**

*One-day standard deviation and correlation statistics based on historical market prices*

Portfolio	Shares held	Price per share, (in USD)	One-day standard deviation, (in %)	Correlation	
				Stock 1	Stock 2
Stock 1	100	100	1	1.00	0.50
Stock 2	100	50	2	0.50	1.00

Table 2 reports that positions in stocks 1 and 2 are USD10,000 and USD5,000, respectively. Estimates of volatility  $\hat{V}$  and the correlation matrix,  $\hat{R}$ , are given by

$$[4] \quad \hat{V} = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix} \text{ and } \hat{R} = \begin{bmatrix} 1.00 & 0.50 \\ 0.50 & 1.00 \end{bmatrix}$$

We can write the covariance matrix,  $\hat{\Sigma}$ , in terms of volatilities and correlations as

$$[5] \quad \hat{\Sigma} = \hat{V}\hat{R}\hat{V}$$

where  $V$  is a matrix of zeros with volatilities,  $\hat{V}$ , along its diagonal. Note that the decomposition as presented in Eq. [5] allows us to **separate changes in volatilities from changes in correlations**. Using Eq. [4] and Eq. [5] we get

$$[6] \quad \hat{\Sigma} = \begin{bmatrix} 0.0001 & 0.0001 \\ 0.0001 & 0.0004 \end{bmatrix}$$

Lastly, as consistent with RiskMetrics, we assume that daily expected returns are zero, so that

$$[7] \quad \hat{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where the element in the first (second) row refers to the expected return on stock 1 (stock 2).

Suppose that we want to measure the Value-at-Risk of this portfolio under the following scenario.

**Stock Scenario 1**

- *Stock prices drop*: The values of stocks 1 and 2 drop by 5 and 15 percent, respectively. To reflect this move, we set the prior mean to

$$[8] \quad \mu_0 = \begin{bmatrix} -0.05 \\ -0.15 \end{bmatrix}$$

## Exploratory stress-scenario analysis with applications to EMU (continued)

- *Market uncertainty increases:* The volatilities of stocks 1 and 2 increase by 25% and 50%, respectively, over their historical values. The prior volatility vector that reflects this change is

$$[9] \quad V_0 = \begin{bmatrix} 0.0128 \\ 0.0329 \end{bmatrix}$$

- *Stock prices lock together:* The correlation between the two stocks increases to 0.85. This results in a prior correlation matrix (between stock 1 and stock 2).

$$[10] \quad R_0 = \begin{bmatrix} 1 & 0.85 \\ 0.85 & 1 \end{bmatrix}$$

which, together with Eq. [9], implies that the prior covariance matrix is

$$[11] \quad \Sigma_0 = \begin{bmatrix} 0.000164 & 0.000359 \\ 0.000359 & 0.001087 \end{bmatrix}$$

We calculate the VaR of this portfolio assuming that stocks follow the model given by Eq. [1] and Eq. [2]. Table 3 provides the VaR estimates with scenario weights equal to 0, 25%, 50%, 75% and 100%.

Table 3

**Scenario-VaR statistics**

*Scenario 1; Portfolio value = \$15,000*

	Scenario weight, (%)	VaR, (in USD) Scenario 1	Percentage change (rel. to history only)
Low (history only)	0	285	0
	25	651	128
Middle	50	1010	254
	75	1364	378
High	100	1714	501

Table 3 shows that if we rely exclusively on historical information (zero weight), the VaR estimate is USD285 or approximately 2% of the current portfolio value; with a 50% weight, VaR is USD1010 (about 7% of the portfolio value); if the event occurs with a weight equal to 100%, then VaR jumps to USD1714, or 11% of the portfolio value.

**Stock Scenario 2**

Next, consider a different scenario where both stock prices fall by 10 percent (relative to the previous day's value — see Eq. [4]) and the volatilities of both stocks increase by 15 percent. We continue to assume that correlations increase (lock together) as they did in the first scenario. In this case the prior mean and covariance matrix are

$$[12] \quad \mu_0 = \begin{bmatrix} -0.10 \\ -0.10 \end{bmatrix} \text{ and}$$

$$[13] \quad \Sigma_0 = \begin{bmatrix} 0.000135 & 0.000229 \\ 0.000229 & 0.000540 \end{bmatrix}$$

## Exploratory stress-scenario analysis with applications to EMU (continued)

Before we present the VaR estimates associated with this scenario, the reader may want to check her intuition and think about which scenario leads to a greater VaR estimate: *Stock Scenario 1* where there is a relatively large move in volatilities, or *Stock Scenario 2* where the volatilities are smaller?

Table 4 reports the VaR estimates for this new scenario for five different weights.

Table 4

**Scenario-VaR statistics**

*Stock Scenario 2; portfolio value = \$15,000*

Scenario weight, (%)	VaR, (in USD), Scenario 2	Percentage change (rel. to history only)
0 (history only)	285	0
25	683	140
50	1079	279
75	1474	417
100	1868	555

A comparison of the VaR estimates reported in Tables 3 and 4 shows that *Stock Scenario 2* leads to marginally higher VaR estimates than *Stock Scenario 1*. This result reflects the increase in the expected fall in value of the larger position (stock 1) from -5% to -10%.

In summary, we have demonstrated how it is possible to generate VaR estimates for two different scenarios with weights 0%, 25%, 50%, 75% and 100%. The model we employ provides us with a straightforward way to investigate how VaR changes with the magnitude and relative importance of a particular scenario. This model is rather flexible in that it can map market scenarios into changes in means, volatilities and correlations.

Although quite simple, *ESSA* has the potential to be very powerful. Two specific cases where this tool can prove particularly useful are:

- investigating how a portfolio's risk profile changes when means, volatilities and/or correlation change.<sup>11</sup> More importantly, this tool allows us to check whether one's intuition about the effect that a particular scenario has on a portfolio's value is accurate or not.
- measuring how the risk of large portfolios changes in response to a particular scenario. Such would be the case with measuring corporate-wide risk, where due to numerous positions across a wide range of asset classes, the so-called 'diversification benefits' are not so obvious and there is little or no intuition about how a particular scenario will affect a portfolio's value.

In the next section we present a more detailed example that underscores the usefulness of this tool.

<sup>11</sup> For example, this tool allows you to address the question, "Given my portfolio, what should I be most concerned about, changes in expected value, volatility shocks or correlation breakdowns?"



**Exploratory stress-scenario analysis with applications to EMU (continued)****1.2 Applications to large portfolios**

In the example presented in Section 1.1, it is rather apparent what effect, if any, a particular scenario would have on the mark-to-market value of that portfolio.<sup>12</sup> There, it is easy to determine whether a gain or loss would result from a particular scenario because we know we are long the two stock positions. For portfolios that contain numerous positions, each of whose value may depend on a different financial price, it is especially difficult to know how sensitive that portfolio is to particular scenarios. Next, we present an example where we demonstrate the impact that four different scenarios have on the value of a hypothetical portfolio consisting of positions in ten (equally weighted) equity positions. We assume that the base currency is USD and, therefore, all risk is measured in terms of US dollars. Table 5 reports the current share price of equity, RiskMetrics one-day volatilities, and size (in USD) of the equity positions.

*Table 5***Equity index portfolio***Closing prices and RiskMetrics one-day volatilities as of March 19, 1998**Portfolio value = \$10,000,000*

	Country	Index	Closing price, (in local ccy)	One-day volatility, (in %)	Position, (in USD)
1	Germany	DAX	4,936.32	1.080	1,000,000
2	Spain	Madrid General	827.06	1.144	1,000,000
3	France	CAC 40	3,688.68	1.132	1,000,000
4	Finland	Helsinki General	4,378.14	1.225	1,000,000
5	Indonesia	Jakarta Composite	504.14	2.893	1,000,000
6	Italy	MiBTel	22,337.00	1.133	1,000,000
7	Japan	Nikkei 225	16,679.02	1.495	1,000,000
8	Korea	Seoul Composite	525.95	3.028	1,000,000
9	Thailand	SET	503.55	2.508	1,000,000
10	Taiwan	Taiwan weighted	8,904.23	1.199	1,000,000

Since we are assuming that risk is measured in USD and positions are held outside of the U.S., each equity position has associated with it a foreign exchange component. Table 6 (page 39) reports the corresponding foreign exchange positions associated with holding equity in countries outside of the US and their associated one-day RiskMetrics volatilities.

<sup>12</sup> That is, whether a scenario would lead to a gain or loss in a portfolio's value. Still, even in that simple illustrative example, the magnitude of the gain or loss may not be obvious.

## Exploratory stress-scenario analysis with applications to EMU (continued)

Table 6

**Foreign exchange positions and volatilities of equity index portfolio***RiskMetrics one-day volatilities as of March 19, 1998*

	Country	Ccy symbol	One-day volatility, (in %)	Position, (in USD)
1	Germany	DEM	0.477	1,000,000
2	Spain	ESP	0.466	1,000,000
3	France	FRF	0.483	1,000,000
4	Finland	FIM	0.479	1,000,000
5	Indonesia	IDR	6.315	1,000,000
6	Italy	ITL	0.463	1,000,000
7	Japan	JPY	0.731	1,000,000
8	Korea	KRW	2.508	1,000,000
9	Thailand	THB	2.391	1,000,000
10	Taiwan	TWD	0.459	1,000,000

As the last column in Table 6 shows, we assume that this portfolio is unhedged and, therefore, its value is completely exposed to foreign exchange risk. The one-day VaR estimate of this unhedged equity portfolio based on RiskMetrics volatilities and correlations for March 19, 1998 is USD280,403<sup>13</sup> or about 2.8% of the portfolio's mark-to-market value.

Let's investigate how the risk of this portfolio changes under four hypothetical market scenarios involving Asian and European currencies and equities. The four scenarios are described as follows:

- **Market Scenario 1:** *Asian currencies weaken against the US dollar.* That is, the currencies of Indonesia, Thailand, Korea and Taiwan all depreciate against the US dollar. These currencies move in lock-step reflecting a substantial increase in correlation. The increase in correlation combined with a rapid depreciation reflects systemic risk.
- **Market Scenario 2:** *Asian currencies weaken (as in Market Scenario 1) and European currencies strengthen relative to the US dollar.* In particular, the deutschemark and french franc increase against the US dollar. Since the one-day RiskMetrics correlations among the European currencies are currently very high (0.98, on average), we do not assume any movements in correlations.
- **Market Scenario 3:** *Asian and European stock prices fall.* Along with the fall in equity prices, we assume that correlation among Asian equity prices, as well as among European equity prices, increase dramatically. We do not, however, make any assumptions about the change in correlation between Asian and European equity prices.
- **Market Scenario 4:** *Asian stock market and currencies fall, European stock markets and currencies rally.* In this case, stock prices in Thailand, Korea and Indonesia fall and rise in Germany, France and Italy. These movements are associated with increased correlations among regional stock prices. Meanwhile, currencies against the USD fall in Asia and rise in Europe (i.e., in Germany and France)

Table 7 (page 40) provides a detailed description of each of the scenarios.<sup>14</sup>

<sup>13</sup> Remember that RiskMetrics assumes that the expected return on all assets over one-day is zero.

<sup>14</sup> Note that these scenarios are purely hypothetical and are simply for demonstrating how *ESSA* works.

## Exploratory stress-scenario analysis with applications to EMU (continued)

Table 7  
Market Scenario description

Scenario	Description	Effect on:	
		Equity	Foreign exchange
1	Asian currencies weaken.	No effect.	1. IDR, THB fall by 5%, and KRW and TWD by 10%, against USD. Correlations among IDR, THB, KRW and TWD increase to 80% and IDR, THB, KRW and THB volatilities increase by 20%, against USD.
2	Asian currencies weaken and European currencies strengthen.	No effect.	1. Same as <i>Market Scenario 1</i> . 2. DEM and FRF increase by 10% against USD (see Appendix B). DEM and FRF volatility against USD increases by 10%.
3	European and Asia stock prices fall.	1. German, Italian and French stock markets fall by 5%. Volatilities increase by 10% and correlations among equity goes to 90%. 2. Stock markets in Thailand, Korea and Indonesia fall by 7%, volatilities go up by 10% and correlations increase to 90%.	No effect.
4	Asian stock market and currencies fall, European stock markets and currencies rally.	1. Same as #2 in <i>Market Scenario 3</i> . 2. German, Italian and French stock markets rise by 5%. Volatilities increase by 10% percent and correlations among equity goes to 90%	1. Same as <i>Market Scenario 1</i> . 2. Same as #2 in <i>Market Scenario 2</i> .

In order to change the correlation structure among financial prices, we employ the recent methodology developed in Finger's article (1997). In that work, Finger presents a technique for changing the correlation among financial prices while preserving the fundamental properties of the correlation matrix. That technique is very important for the results presented in this article since it allows us to meaningfully quantify increases and decreases in correlations among financial prices that result from economic and financial events.

Exploratory stress-scenario analysis with applications to EMU (continued)

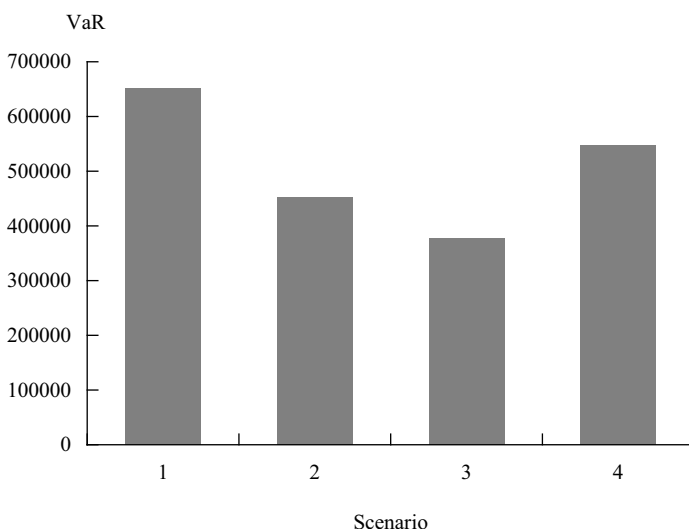
Table 8 reports the VaR estimates under each of the four market scenarios.

Table 8  
**VaR statistics in USD**  
Unhedged portfolio; relative (to RiskMetrics) percentage change in parentheses

Scenario weight, (%)	RiskMetrics VaR, (in USD)	Scenario Var, (in USD)			
		1	2	3	4
0	280,403	280,403 (0.00)	280,403 (0.00)	280,403 (0.00)	280,403 (0.00)
25	NA	374,624 (33.6)	324,778 (15.8)	305,028 (8.78)	350,520 (25.01)
50	NA	467,685 (66.8)	367,974 (31.23)	329,343 (17.45)	417,965 (49.06)
75	NA	559,773 (99.63)	410,184 (46.3)	353,377 (26.02)	483,342 (72.37)
100	NA	651,030 (132.2)	451,552 (61.0)	377,154 (34.50)	547,059 (95.10)

The results reported in Table 8 show that a weakening of Asian currencies alone (*Market Scenario 1*) leads to the greatest increase in VaR (maximum 132% increase over RiskMetrics VaR), whereas a drop in Asian and European stock prices (*Market Scenario 3*) produces the smallest increases in VaR (maximum 34.5% increase over RiskMetrics VaR). Chart 4 illustrates the Scenario-VaR estimates for the four market scenarios when the scenario weight is 100%.

Chart 4  
**Scenario-VaR estimates**  
Weight = 100%



Next, we demonstrate how *ESSA* can be used to assess the relationship between position size and VaR estimates associated with different market scenarios. We assume that we hold the same equities as reported

## Exploratory stress-scenario analysis with applications to EMU (continued)

in Table 5, but we have hedged a large portion of our foreign exchange exposure in Asia. Table 9 reports the hedged foreign exchange positions.

Table 9

**New foreign exchange positions for hedged portfolio**

	Country	Ccy symbol	Position, (in USD)
1	Germany	DEM	1,000,000
2	Spain	ESP	1,000,000
3	France	FRF	1,000,000
4	Finland	FIM	1,000,000
5	Indonesia	IDR	500,000
6	Italy	ITL	1,000,000
7	Japan	JPY	750,000
8	Korea	KRW	250,000
9	Thailand	THB	500,000
10	Taiwan	TWD	750,000

Table 10 reports the VaR estimates under each of the four scenarios for the hedged portfolio.

Table 10

**VaR statistics in USD**

*Hedged portfolio; relative (to RiskMetrics) percentage change in parentheses*

Scenario weight, (%)	RiskMetrics VaR, (in USD)	Scenario VaR, (in USD)			
		1	2	3	4
0	216,562	216,562 (0.00)	216,562 (0.00)	216,562 (0.00)	216,562 (0.00)
25	NA	261,525 (20.7)	211,686 (-2.3)	241,972 (11.7)	237,798 (9.8)
50	NA	306,246 (41.4)	206,558 (-4.6)	266,925 (23.3)	257,647 (18.9)
75	NA	350,749 (61.9)	201,203 (-7.1)	291,476 (34.6)	276,380 (27.62)
100	NA	395,051 (82.4)	195,640 (-9.7)	315,670 (45.7)	294,187 (35.84)

Table 10 shows that *Market Scenario 2* (Asian currencies weaken and European currencies strengthen) actually reduces the risk of the portfolio, compared to RiskMetrics. That is, the more weight the scenario is given, the lower the VaR estimate. This result is due to that fact that we attached positive expected returns to the relatively large European positions. This is a good example of how *ESSA* could be used to quantify the effect that different hedging strategies have on VaR under different stress scenarios.

More generally, the results reported in Table 10 show that overall, the VaR estimates for the hedged portfolio fall well below those of the unhedged portfolio (Table 9). To facilitate the comparison between the VaR estimates from the hedged and unhedged portfolio, Table 11 (page 43) provides the relative

## Exploratory stress-scenario analysis with applications to EMU (continued)

percentage change between the VaR estimates of the hedged and unhedged portfolio. Negative table entries correspond to the case where the risk from the unhedged portfolio is larger than the risk from the hedged portfolio.

Table 11

**Relative percentage difference between hedged and unhedged portfolios**

Table entries =  $100 \times (VaR \text{ hedged} - VaR \text{ unhedged}) / VaR \text{ unhedged}$

Scenario weight, (%)	Scenario			
	1	2	3	4
0	-23	-23	-23	-23
25	-30	-35	-21	-32
50	-35	-44	-19	-38
75	-37	-51	-18	-43
100	-39	-57	-16	-46

The conclusions drawn from Table 11 are clear: in US dollar terms, the unhedged portfolio is more risky (i.e., it has a larger VaR estimate) than its hedged counterpart.

**2. EMU-related stress scenarios**

In the previous two sections we developed an infrastructure and provided examples to assess the impact that stress-scenarios have on a portfolio's VaR. *ESSA* may prove to be particularly useful when it is thought that VaR estimates that rely exclusively on historical market prices, such as those provided by RiskMetrics, are effectively useless. The formation of EMU presents a potential situation where *ESSA* can be applied to study the effect of potential stress scenarios on portfolio VaR. In this section, we demonstrate how *ESSA* can be used to 'play out' potential **EMU-related stress scenarios** and determine how such scenarios impact VaR estimates.

We examine how market scenarios that may result from EMU—so-called EMU-related stress scenarios—affect the VaR of a USD-based portfolio consisting of fixed income and equity positions in Europe and Asia. Note that we take no particular stand on the likelihood or possibility of any of the EMU-related scenarios. Rather, our primary purpose is to illustrate how *ESSA* can be used to analyze the effect that different market scenarios have on a portfolio's VaR.

Table 12 presents the type and position amount for our sample portfolio.

Table 12

**Sample EMU portfolio position values in USD**

Equity and fixed income positions (in 000s)

Portfolio value = \$15,000,000

Positions	Europe					Asia	
	Germany	France	Italy	Spain	UK	Japan	Singapore
equity	750	1,200	1,000	500	1,500	1,000	500
1-year deposit	250	500	1,000	500	250	200	500
10-year gov't bond	1,500	500	750	500	1,200	150	750
foreign exchange	2,500	2,200	2,750	1,500	2,950	1,350	1,750

The foreign exchange positions reported in Table 12 result from holding equity and fixed income positions outside of the US.

**Exploratory stress-scenario analysis with applications to EMU (continued)**

The literature on EMU is replete with market scenarios (e.g., ITL drops 15% against DEM) that may result prior to and during the formation of EMU. It seems that the type of EMU-related stress scenario is limited only by the imagination of the person who created that scenario. We focus on scenarios that may apply prior to January 1, 1999 in a way as to affect foreign exchange rates, equity prices and interest rates. The scenarios are as follows:

- **EMU Scenario 1:** Transition to EMU goes smoothly. Heading into January 1999, the euro has all the markings of becoming a low inflation, low volatility currency. In this case, history is the best guide to the future.
- **EMU Scenario 2:** Investors perceive EMU as potentially not starting. DEM falls 10% against USD and the ITL falls 15% against the DEM. In addition long-term rates in EMU countries increase 200 basis points.
- **EMU Scenario 3:** Stock markets in EMU countries fall by 25% and their currencies fall 15% against USD. To capture systemic risk (contagion), we assume that equity price correlations among EMU countries increase to 90%.
- **EMU Scenario 4:** Stock markets in EMU countries fall by 25% and their currencies fall 15% against USD. We assume that equity price correlations among EMU countries fall to 0%.
- **EMU Scenario 5:** Large price swings occur as uncertainty creeps into the market doubling market volatilities among EMU equity, fixed income and foreign exchange positions. Correlations remain at their historical levels.

Before we present the results that show how each scenario affects VaR, some comments are in order pertaining to *EMU Scenario 2*, which specifies a movement in the cross exchange rate ITL/DEM. Since we are assuming that the base currency is US dollars, we have to translate the change in ITL/DEM into changes in USD-based currencies. In Appendix B, we demonstrate how a 15% increase in ITL/DEM and a 10% decrease in USD/DEM implies that the lira falls 25% against the US dollar.

In addition, *EMU Scenario 2* expresses a movement in interest rates (basis points). RiskMetrics, however, uses price volatilities and correlations (i.e., volatilities and correlations based on bond prices) to compute VaR. Therefore, in order to determine what effect this scenario will have on our VaR estimate, we must convert the change in basis points to an equivalent change in the expected return on bond prices. Appendix C explains how we convert the expected change in basis points to the expected return on bond prices.

Table 13 (page 45) presents the VaR estimates for the five EMU scenarios with scenario weights equal to 0%, 25%, 50%, 75%, and 100%.

Exploratory stress-scenario analysis with applications to EMU (continued)

Table 13

**Scenario-VaR statistics in USD**

Sample EMU portfolio; relative (to RiskMetrics) percentage change in parentheses

Scenario weight, (%)	Scenario, (in USD)				
	1	2	3	4	5
0	139,372 (0.00)	139,372 (0.00)	139,372 (0.00)	139,372 (0.00)	139,372 (0.00)
25	NA	536,463 (284.7)	692,359 (396)	685,574 (391)	185,057 (32)
50	NA	933,553 (569.2)	1,245,325 (793)	1,231,579 (783)	221,511 (59)
75	NA	1,330,643 (854.9)	1,798,271 (1190)	1,777,361 (1175)	252,762 (81)
100	NA	1,727,733 (1139)	2,351,197 (1586)	2,322,890 (1566)	280,553 (101)

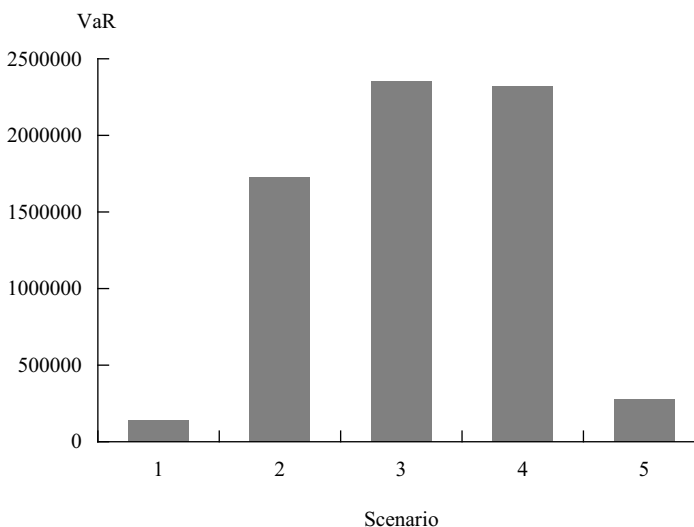
The results from Table 13 show, not surprisingly, that *EMU Scenario 3* leads to the largest VaR estimate. Interestingly, there is little difference between the VaR estimates that are based on correlations of 90% (*EMU Scenario 1*) and 0% (*EMU Scenario 2*) among EMU stock markets. The VaR estimates associated with *EMU Scenario 5* show that even if volatilities double across all EMU positions, the increase in VaR is relatively small. Obviously, these VaR estimates depend on the positions we hold.

Chart 5 illustrates the relative differences between the VaR estimates under the five EMU scenarios.

Chart 5

**EMU-related Scenario-VaR estimates**

Weight = 100%



Finally, in this section and in the last, we demonstrated how risk managers can assess the impact that particular scenarios have on the VaR of a set of positions that do not involve any options. When options are



**Exploratory stress-scenario analysis with applications to EMU (continued)**

introduced, the methodology discussed above is the exact same; the only difference is how it is applied. In the next section, we discuss how to include options into *ESSA*.

**3. Nonlinear risk**

Two popular methodologies that industry professionals currently employ to measure market risk of option positions are Full Monte Carlo Simulation (FMCS) and delta/gamma (DG). Whereas FMCS relies exclusively on full valuation, in DG, an analytical expression based on an option's underlying 'greeks' is used to approximate the value of an option. Next, we demonstrate how to use both methodologies in *ESSA*.

**3.1 Full Monte Carlo Simulation (FMCS)**

Standard FMCS requires that we simulate price distributions using some estimate of the mean and covariance matrix of market returns. The simulation is done in three steps:

1. Generate draws of correlated market returns from the multivariate normal distribution with a mean  $m$  and covariance matrix,  $C$ .
2. Convert these market returns to respective prices and rates.
3. Value the option positions at the simulated prices and rates.

To implement *ESSA*, we simply use the weighted mean and covariance matrices that are defined in Eq. [2] to simulate market returns and market prices/rates that are consistent with market scenarios. These simulated prices and rates are then used in the valuation process.

In situations where there is concern about the risk associated with movements in implied volatility, we can change implied volatilities in the same manner that we adjusted historical volatilities to reflect market uncertainty. Option positions would then be revalued under these new, adjusted implied volatilities.

**3.2 Delta/Gamma (DG) methodology**

According to the DG methodology, the price of an option can be approximated by a combination of its delta, gamma, and changes in underlying market variables. This approximation may be improved by including other 'greeks' such as the option's theta (time) and vega (implied volatility). Next, we show how it is straightforward to use DG in *ESSA*. To keep matters simple, we focus exclusively on delta and gamma, keeping in mind that it is relatively simple to account for theta and vega as well.<sup>15</sup>

In the standard DG framework, we **expand around last period's price** such that the value of an option at time  $t$  can be written as:

$$[14] \quad V_t \approx V_{t-1} + \Delta_{t-1}(S_t - S_{t-1}) + 0.5\Gamma_{t-1}(S_t - S_{t-1})^2$$

<sup>15</sup> A complete explanation of the DG approach can be found in Section 6.3 of the *RiskMetrics™—Technical Document*, December 1996, 4th edition.

## Exploratory stress-scenario analysis with applications to EMU (continued)

where

$V_t$  and  $V_{t-1}$  are tomorrow and today's option value, respectively.

$S_t$  and  $S_{t-1}$  are tomorrow and today's price of the underlying asset, respectively.

$\Delta_{t-1}$  and  $\Gamma_{t-1}$  are today's delta and gamma, respectively. The values of these parameters are based on today's spot price,  $S_{t-1}$ .

After some manipulation, it is possible to write the return on the option in terms of returns on the underlying and adjusted delta and gamma, i.e.,

$$[15] \quad r_{option} \approx \tilde{\Delta}_{t-1} r_t + 0.5 \tilde{\Gamma}_{t-1} r_t^2$$

where

$r_{option}$  is the return on the option.

$r_t$  is the return on the underlying asset.

$\tilde{\Delta}_{t-1}$  and  $\tilde{\Gamma}_{t-1}$  are the adjusted delta and gamma, respectively. (See Section 6.3 of the *RiskMetrics™—Technical Document* [4th edition].)

Now, in *ESSA* we are interested in measuring deviations from stress values of the spot price. Therefore, instead of expanding around the current spot price,  $S_{t-1}$ , **we expand around the spot price implied by a particular scenario**  $S_s$  to get

$$[16] \quad V_{s,t} \approx V_{s,t-1} + \Delta_s (S_t - S_s) + 0.5 \Gamma_s (S_t - S_s)^2$$

where, now

$V_{s,t}$  and  $V_{s,t-1}$  are tomorrow and today's option value, respectively, evaluated at the stress price  $S_s$ .

$\tilde{\Delta}_s$  and  $\tilde{\Gamma}_s$ , are the adjusted delta and gamma, respectively, evaluated at the stress price  $S_s$ .

In terms of Eq. [16], the return on the option,  $r_{s,option}$ , under the stress scenario becomes

$$[17] \quad r_{s,option} \approx \tilde{\Delta}_s r_{s,t} + 0.5 \tilde{\Gamma}_s r_{s,t}^2$$

where

$r_{s,t}$  represents the returns generated from the distribution described by Eq. [1] and Eq. [2]

$\tilde{\Delta}_s$  and  $\tilde{\Gamma}_s$ , are the adjusted delta and gamma, respectively, evaluated at the stress price  $S_s$ .

Using Eq. [17] we can calculate the return on as many options as necessary and apply the methods set forth in Section 6.3 of the *RiskMetrics™—Technical Document* (4th edition) that explains how to implement DG.

**Exploratory stress-scenario analysis with applications to EMU (continued)****4. Conclusions**

As the formal establishment of phase one of EMU nears, risk managers are concerned about how its developments will affect, if at all, the stability of financial markets. Therefore, it is important that risk managers have a tool that allows them to study how an event such as EMU, and its repercussions, may affect the value of their portfolio. This article presents a tool to perform exploratory stress scenario analysis, which we refer to as *ESSA*. *ESSA* allows risk managers to convert their views on potential market scenarios into statistical parameter estimates that enter the VaR calculation. We provided three sets of scenarios of how *ESSA* can be applied, including an application to EMU-related stress scenarios. Our study shows that when it comes to analyzing the impact that scenarios have on simple positions, changes in the mean values have the greatest impact on VaR estimates followed by volatilities and correlations. Finally, we established how this methodology can be applied to option positions as well.

Exploratory stress-scenario analysis with applications to EMU (continued)

**Appendix A**

In this appendix<sup>16</sup> we present a formal methodology for incorporating prior information into a statistical model. The model is based on a standard application of Bayesian statistics.

Assume that returns,  $R_t$ , follow a multivariate normal distribution with density function given by

$$[A.1] \quad f_n(R_t | \tilde{\mu}, \tilde{\Sigma})$$

In the Bayesian paradigm, it is assumed that the parameters  $\tilde{\mu}$  and  $\tilde{\Sigma}$  are random variables and, therefore, have their own statistical distributions. These distributions are introduced through so-called prior information. The primary method for introducing prior information into any Bayesian decision rule is through specification of a natural conjugate prior. The Normal-Wishart probability distribution is the conjugate prior to the multivariate normal probability distribution. We assume that the Normal-Wishart conjugate prior with prior parameters  $\Sigma_0^{-1}$ ,  $\nu$ ,  $\mu_0$ , and  $\tau$ , is given by

$$[A.2] \quad f_{nw}(\tilde{\mu}, \tilde{\Sigma}^{-1} | \Sigma_0^{-1}, \nu, \mu_0, \tau) = f_n(\tilde{\mu} | \mu_0, \tilde{\Sigma} / \tau) \times f_w(\tilde{\Sigma}^{-1} | \Sigma_0^{-1} \frac{1}{\nu}, \nu)$$

where

$f_n(\tilde{\mu} | \mu_0, \tilde{\Sigma} / \tau)$  represents the multivariate normal probability distribution for the parameter  $\mu$  with mean  $\mu_0$  and covariance matrix  $\tilde{\Sigma} / \tau$ .

$f_w(\tilde{\Sigma}^{-1} | \frac{1}{\nu} \Sigma_0^{-1}, \nu)$  represents the Wishart distribution. That is, the inverse of the covariance matrix,  $\tilde{\Sigma}^{-1}$ , is assumed to follow a Wishart distribution with parameters  $\frac{1}{\nu} \Sigma_0^{-1}$  and  $\nu$ .

$f_{nw}(\tilde{\mu}, \tilde{\Sigma}^{-1} | \Sigma_0^{-1}, \nu, \mu_0, \tau)$  represents the joint probability density function for  $\tilde{\mu}$  and  $\tilde{\Sigma}^{-1}$ .

The prior parameters  $\nu$  and  $\tau$  determine the strength of belief in prior values  $\Sigma_0^{-1}$  and  $\mu_0$ . Given the assumption of a Normal-Wishart conjugate prior, the predictive density for returns,  $R_t$ ,  $g(R_t)$ , is multivariate student- $t$  distributed according to

$$[A.3] \quad f_s(R_t | \mu, \Sigma, \nu + T)$$

with  $\nu + T$  degrees of freedom where  $T$  represents the amount of sample information (i.e., the number of historical returns). Posterior estimates  $\mu$  and  $\Sigma$  follow from the posterior distribution of  $\tilde{\mu}$  and  $\tilde{\Sigma}$ , and are given by

$$[A.4] \quad \mu = \omega_\mu \mu_0 + (1 - \omega_\mu) \hat{\mu}$$

$$[A.5] \quad \Sigma = \left[ \frac{\nu + T}{\nu + T - 2} \right] \left[ 1 + \frac{1}{\tau + T} \right] \left[ \Sigma_0 \omega_\nu + \hat{\Sigma} (1 - \omega_\nu) + \omega_\tau \left( \frac{T}{\nu + T} \right) (\hat{\mu} - \mu_0)(\hat{\mu} - \mu_0)^T \right]$$

where  $\omega_\tau = \tau / (\tau + T)$  and  $\omega_\nu = \nu / (\nu + T)$

<sup>16</sup> This appendix is based on Frost and Savarino (1986).

**Exploratory stress-scenario analysis with applications to EMU (continued)**

For any given amount of sample information  $T$ , strength-of-belief weights  $\omega_\tau$  are determined, respectively, by prior parameters  $\tau$  and  $\nu$ . From Eq. [A.4], posterior estimate  $\mu$  of the mean vector  $\mu$  will more closely approximate prior value  $\mu_0$ , the larger the strength of belief weight  $\omega_\tau$ . Similarly, from Eq. [A.5], for any given value of  $T$  and  $\omega_\tau$ , the prior  $\Sigma_0$  will be provided a larger weight in determining posterior estimate  $\Sigma$ , the larger the value of the strength of belief weight  $\omega_\nu$ .

Now, note that Eq. [A.4] is the same as the expression for the mean given in Eq. [2]. The expression for the posterior covariance given by Eq. [A.5] is approximately equal to covariance term given in Eq. [2] for large values of  $T$ .

**Exploratory stress-scenario analysis with applications to EMU (continued)****Appendix B**

In this appendix we establish the relationship between the expected return on the USD/DEM and USD/ITL exchange rates and the ITL/DEM cross-exchange rates in exchange rate. In the *RiskMetrics™—Technical Document* (4th edition, Section 8.4, pp. 183-184), we explain how to write the return on the ITL/DEM ( $r_1$ ) exchange rate as the difference between the return on the USD/DEM exchange rate ( $r_2$ ) and the USD/ITL exchange rate ( $r_3$ ), i.e.,

$$[\text{B.1}] \quad r_1 = r_2 - r_3$$

It follows from Eq. [B.1] that the expected value of the return on the ITL/DEM exchange rate, ( $\mu_1$ ), is the difference between the expected return on USD/DEM ( $\mu_2$ ) and USD/ITL ( $\mu_3$ ), i.e.,

$$[\text{B.2}] \quad \mu_1 = \mu_2 - \mu_3$$

Hence, if  $\mu_1 = 0.15$  and  $\mu_2 = -0.10$  then the expected return on the USD/ITL exchange rate is equal to  $-0.25$ , or  $\mu_3 = -0.25$ .

## Exploratory stress-scenario analysis with applications to EMU (continued)

**Appendix C**

In this appendix we explain how to convert a statement on the number of basis points interest rates will move to the expected return on the price of a corresponding zero-coupon bond.

The price of a  $\tau$ -period zero-coupon bond at time  $t$  is given by the expression

$$[C.1] \quad P_t = \exp(-y_t \tau)$$

where  $\tau$  is measured in years, and  $y_t$  is the annualized zero-coupon interest rate and  $\exp(x) = e^x$ .

Now, the time  $t$  one-day price return on the bond is

$$[C.2] \quad r_t = \ln(P_t) - \ln(P_{t-1})$$

which, using Eq. [C.1], can be written as

$$[C.3] \quad r_t = -\tau(y_t - y_{t-1})$$

The expected price return at time  $t-1$ ,  $\mu$ , is then given by

$$[C.4] \quad \mu = E_{t-1}(r_t) = -\tau(E_{t-1}(y_t) - y_{t-1})$$

where  $E_{t-1}(x)$  denotes the mathematical expectation of  $x$  at time  $t-1$ .

Now, suppose that under a particular scenario we expected (at  $t-1$ ) that the interest rate  $y_t$  increases by  $n$  basis points. Since one basis point is equal to 0.01%, or 0.0001, we can write the expected value of  $y_t$ ,  $E_{t-1}(y_t)$ , as

$$[C.5] \quad E_{t-1}(y_t) = y_{t-1} + 0.0001 \times n$$

where  $n$  denotes the number of basis points increase ( $n > 0$ ) or decrease ( $n < 0$ ). So, for example if the current interest rate is 5.04% and we expect interest rates to increase by 3 basis points, we have  $y_{t-1} = 0.0504$  and  $n = 3$ , so that

$$[C.6] \quad \begin{aligned} E_{t-1}(y_t) &= 0.0504 + 0.0001 \times 3 \\ &= 0.0507 \end{aligned}$$

Eq. [C.4] and Eq. [C.5] imply that the expected price return can be written as a function of the number of basis points change, i.e.,

$$[C.7] \quad \mu = -\tau(0.0001 \times n)$$

Eq. [C.7] allows us to map directly from the change in the number of basis points under a particular scenario to the expected price return on the bond. For example, a 5-basis point decrease for a 10-year zero-coupon bond implies a 0.5% increase in the price return on the bond. Note that the longer the bond's maturity, the more sensitive the bond price is to changes in the expected interest rate.

**Exploratory stress-scenario analysis with applications to EMU (continued)****References**

Finger, Christopher. 1997. "A methodology to stress correlations." *RiskMetrics Monitor*, Fourth Quarter, 1997.

Frost, P.A. and J.E. Savarino. 1986. "An Empirical Bayes Approach to Efficient Portfolio Selection," *Journal of Financial and Quantitative Analysis*, Vol. 21, No. 3, (September), pp. 293-305.

Kupiec, Paul. 1998. "Stress Testing in a Value-at-Risk Framework." J.P. Morgan risk management research working paper.

Zangari, Peter. 1996. "An improved methodology for measuring VaR," *RiskMetrics Monitor*, Second Quarter, 1996.