# Black Swans and Market Timing: How Not To Generate Alpha

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**Abstract**

*Do investors obtain their long term returns smoothly and steadily over time, or is their long term performance largely determined by the return of just a few outliers? How likely are investors to successfully predict the best days to be in and out of the market? The evidence from 15 international equity markets and over 160,000 daily returns indicates that a few outliers have a massive impact on long term performance. On average across all 15 markets, missing the best 10 days resulted in portfolios 50.8% less valuable than a passive investment; and avoiding the worst 10 days resulted in portfolios 150.4% more valuable than a passive investment. Given that 10 days represent less than 0.1% of the days considered in the average market, the odds against successful market timing are staggering.*

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## 1. Introduction

> "*Measures of uncertainty that are based on the bell curve simply disregard the possibility, and the impact, of sharp jumps … Using them is like focusing on the grass and missing out on the (gigantic) trees. Although unpredictable large deviations are rare, they cannot be dismissed as outliers because, cumulatively, their impact is so dramatic.*" Taleb (2007).

Consider an investor that put $100 in the Dow Jones Industrial Average on the closing bell of Dec/31/1986. Through Oct/16/1987, when the Dow closed at 2246.7, this investor obtained an 18.5% return, turning his initial $100 into $118.5. Then, in a single day, the market tumbled 22.6%, and the investor's $118.5 were reduced to $91.7. The return of 201 trading days more than wiped out in a single day.[1] It took the Dow 320 trading days to get past the level of Oct/16/1987; on Jan/24/1989 it closed at 2256.4.

Black Monday, as Oct/19/1987 became known, was not just another day; it was the single worst day (in percentage terms) in the Dow's history and therefore unique. But what is neither unique nor unusual is that a few large daily swings can more than overturn the return of a

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[1] These figures do not account for dividends, but doing so does not change the essence of the story. Accounting for dividends, the return between Dec/31/1986 and Oct/16/1987 was 21.3%, which turned the $100 investment into $121.3. The 22.6% fall on Oct/19/1987 turned that stake into $93.9, still more than wiping out the return of the previous 201 days.

portfolio obtained over a long period of time. Interestingly, under the widely used and abused assumption of normality, this should happen very infrequently, if at all. And yet the evidence shows that these events happen *far* more often than would be expected under this assumption.

This article, however, is not about testing for the normality of selected assets, as is done in the pioneering work of Mandelbrot (1963) and Fama (1965). Nor is it about finding the distribution that best fits the returns of selected assets, as is done in Aparicio and Estrada (2001); nor about characterizing the tail behavior of selected assets, as is done in Jansen and de Vries (1991) and Longin (2005). The ultimate goal of this article, instead, is to quantify the impact of outliers on long term performance. Do investors obtain their long term returns smoothly and steadily over time, or is their long term performance largely determined by the return of just a few outliers? Are investors likely to successfully predict the best days to be in and out of the market? Should investors attempt to time the market? Those are the ultimate issues addressed here.

The evidence, based on more than 160,000 daily returns from 15 international equity markets, is clear: Outliers have a massive impact on long term performance. On average across all 15 markets, missing the best 10 days resulted in portfolios 50.8% less valuable than a passive investment; and avoiding the worst 10 days resulted in portfolios 150.4% more valuable than a passive investment. Given that 10 days represent, in the average market, less than 0.1% of the days considered, the odds against successful market timing are staggering. Hence, of the countless strategies that academics and practitioners have devised to generate alpha, market timing does not seem to be the one most likely to succeed.

The rest of the article is organized as follows. Section 2 introduces the issue at stake and discusses the concept of black swans, the assumption of normality, and the impact of short term swings on long term performance. Sections 3 and 4 discuss the evidence, the former focusing on the Dow and the latter on 15 international equity markets. Section 5 revisits the international evidence by focusing on the more recent 1990-2006 period. Finally, section 6 concludes with an assessment and some final thoughts.

## 2. The Issue

This section introduces the issue at stake, first, by briefly discussing the concept of black swans which, although informally defined, is at the heart of the issues addressed here. Then it briefly discusses the widely-used assumption of normality, which is closely related to the issues addressed here. And finally, it concludes with a brief discussion of previous research on the impact of outliers on long term performance.

### *2.1. What Is A Black Swan?*

Taleb (2007) defines a black swan as an event with three attributes: 1) It is an outlier, lying outside the realm of regular expectations because nothing in the past can convincingly point to its occurrence; 2) it carries an extreme impact; and 3) despite being an outlier, plausible explanations for its occurrence can be found after the fact, thus giving it the appearance that it can be explainable and predictable. In short, then, a black swan has three characteristics: Rarity, extreme impact, and retrospective predictability.[2]

Now consider Black Monday. Between inception on May/26/1896 and Oct/16/1987, the Dow had only twice in its whole history fallen by more than 10% in one day. This happened on back-to-back days in the midst of the crash of 1929; on Oct/28/1929 and Oct/29/1929 the Dow fell 12.8% and 11.7%. But nothing in the 90+ years of history of the Dow pointed out to the possibility of a fall of the magnitude observed on Oct/19/1987. And yet, the unexpected and inconceivable did happen. Black Monday was an extremely rare event; it did have a very significant impact on investors' portfolios; and, as discussed by Haugen (1999) and others, many and varied stories were advanced to explain it ex-post. In short, Black Monday was a black swan.

As discussed below, daily swings in the markets do not have to be so dramatic to have a substantial impact on long term performance. For this reason, the focus of this article is on 'large' daily swings, as informally defined below. And although some attention is paid to daily returns more than three standard deviations away from the mean, as well as to the best and worst 10, 20, and 100 daily returns, no attempt is made here to formally define a black swan.

### *2.2. The Normality Assumption*

As mentioned above, it is not the goal of this paper to test for the normality of selected assets. Still, whether daily returns follow a normal distribution underlies the discussion, simply because the large daily swings in which this article focuses have a negligible probability of occurring under the assumption of normality.

Note that, from a theoretical point of view, this assumption is questionable if information does not arrive linearly to the market or, even if it does, if investors do not react linearly to its arrival. Peters (1991) argues that investors may ignore information until trends are in place and eventually react in a cumulative fashion to all the information ignored up to that point. If that is the case, then returns would not be expected to be normally distributed.

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[2] The term black swan is often informally used as a metaphor for something very rare. Its origin stems from the fact that, as far as the Western world was concerned, the hypothesis that all swans were white was an unquestionable statement supported by countless sightings of white swans over many centuries. That belief changed in a single moment, when the Dutch explorer Willem de Vlamingh recorded a first sighting of black swans in the West coast of Australia in January, 1697. It took this one sighting to invalidate a belief supported by centuries of evidence.

From an empirical point of view, Mandelbrot (1963) argued in a pathbreaking article that cotton price changes can be characterized by a stable Paretian distribution with a characteristic exponent less than 2, thus exhibiting fat tails and infinite variance. He tested the infinite-variance hypothesis by computing the variance of a large number of samples of cotton price changes and found that the variances did not converge to any limiting value. Rather, they evolved in an erratic fashion, as would be expected under the infinite-variance hypothesis.

Fama (1965) subsequently tested Mandelbrot's (1963) hypothesis on the 30 stocks of the Dow Jones Industrial Average and confirmed that a stable Paretian distribution with a characteristic exponent less than 2 describes their returns better than the normal distribution. Since then, the normality of stock returns, particularly at the daily frequency, has been rejected in favor of fat-tailed distributions in a wide variety of markets, assets, and time periods; see Aparicio and Estrada (2001), among many others. The evidence discussed in this article also points strongly against the assumption of normally distributed daily returns.

Finally, Mandelbrot and Hudson (2005) propose to simply abandon the normality assumption and replace it with a fractal view of risk, ruin, and reward. In their proposed framework, large swings follow a power law and are therefore far more likely (and clustered) than what a normal distribution would predict.

### *2.3. The Impact of Large Swings on Portfolio Performance*

The idea that outliers are far more often observed than what the normality assumption would predict is not new. Quantifying the impact of these outliers on long term performance, however, is a more recent development. Mauboussin (2006) argues that over the Jan/3/1978-Oct/31/2005 period the S&P-500 delivered a mean annual return of 9.6%; excluding the best 50 days (out of over 7,000) lowers the mean return to 2.2%, and excluding the worst 50 days increases the mean return to 18.4%.[3]

Browne (2007) discusses a study by Sanford Bernstein showing that, during the 1926-93 period, the returns of the U.S. stock market in the best 60 months (7% of the time) averaged 11% whereas the returns of the rest of the months (93% of the time) averaged 0.01%. He concludes that finding the way to reliably predict the 7% of the time that stocks do well is a 'daunting task,' and that the real danger is not to be in the market when big moves occur. He also discusses a study by American Century Investments showing that $10,000 invested in the U.S. stock market in 1990 turned into $51,354 by 2005, but missing the best 10, 30, and 50 days would have reduced the terminal wealth to $31,994, $15,730, and $9,030.

[3] None of these figures account for dividends.

## 3. Evidence (I): The Dow

In order to assess the impact of outliers on long term performance, we will focus first on the U.S. market, and in particular in the Dow Jones Industrial Average index. In the next section we will discuss the evidence from other international markets, as well as for the S&P-500. The results of the analysis of the Dow's daily returns between the beginning of 1900 and the end of 2006 are summarized in Exhibit 1.

**Exhibit 1: The Dow Jones Industrial Average**

This exhibit shows information based on the series of daily returns of the Dow Jones Industrial Average index. Panel A shows the minimum (Min) and maximum (Max) return; arithmetic (AM) and geometric (GM) mean return; standard deviation (SD); coefficients of skewness (Skw) and kurtosis (Krt); and coefficients of standardized skewness (SSkw) and standardized kurtosis (SKrt). Panel B shows the expected (Exp) and observed (Obs) number of daily returns three SDs below and above AM; the ratio between the number of these observed and expected returns; and the total number of expected (TE) and observed (TO) returns more than three SDs away from the mean. Panel C shows the arithmetic mean return for the whole sample (All); the mean return of the best 10, 20, and 100 days (B10, B20, and B100); the mean return of the worst 10, 20, and 100 days (W10, W20, and W100); and the number of SDs away from AM these last six magnitudes are. Panel D shows the terminal value of $100 invested on Dec/31/1899 and held passively through Dec/31/2006 (TV100), not including dividends; such terminal value without being invested during the best 10, 20, and 100 days (–B10, –B20, and –B100); such terminal value without being invested during the worst 10, 20, and 100 days (–W10, –W20, and –W100); the percent changes of these last six terminal values with respect to TV100; and the mean annual compound return (MACR) in all these scenarios. 'Exp' figures are rounded to the nearest integer. Returns account for capital gains but not for dividends. All magnitudes calculated over the Jan/1/1900–Dec/31/2006 period.

*Panel A: Summary Statistics*

| Years | Days | Min | Max | AM | GM | SD | Skw | Krt | SSkw | SKrt |
|---|---|---|---|---|---|---|---|---|---|---|
| 107 | 29,190 | –22.61% | 15.34% | 0.02% | 0.02% | 1.07% | –0.2 | 19.2 | –15.9 | 668.6 |

*Panel B: Outliers*

| AM–3·SD | Exp | Obs | Ratio | AM+3·SD | Exp | Obs | Ratio | TE | TO | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| –3.17% | 39 | 253 | 6.4 | 3.22% | 39 | 208 | 5.3 | 79 | 461 | 5.8 |

*Panel C: Mean Returns*

| | All | B10 | B20 | B100 | W10 | W20 | W100 |
|---|---|---|---|---|---|---|---|
| AM | 0.02% | 11.10% | 9.37% | 5.92% | –10.46% | –8.73% | –5.87% |
| SDs from AM | | 10.4 | 8.8 | 5.5 | 9.8 | 8.2 | 5.5 |

*Panel D: Terminal Values*

| | TV100 | –B10 | –B20 | –B100 | –W10 | –W20 | –W100 |
|---|---|---|---|---|---|---|---|
| TV | $25,746 | $9,008 | $4,313 | $83 | $78,781 | $162,588 | $11,198,734 |
| Change | N/A | –65.0% | –83.2% | –99.7% | 206.0% | 531.5% | 43,396.8% |
| MACR | 5.3% | 4.3% | 3.6% | –0.2% | 6.4% | 7.2% | 11.5% |

As shown in panel A, over the 29,190 trading days (107 years) of the Dow's history considered in the exhibit, the daily (arithmetic and geometric) mean return and standard deviation were 0.02% and 1.07%. The maximum and minimum daily returns were 15.34% and −22.61%, the latter on Black Monday. The coefficients of standardized skewness and kurtosis

indicate a significant degree of (negative) skewness and kurtosis, both of which suggest a very clear departure from a normal distribution.[4,5]

Panel B shows that –3.17% and 3.22% are the limits of the interval three standard deviations around the (arithmetic) mean return. As is well known, under a normal distribution the area within this interval is 99.73%, and the area outside it 0.27%. Therefore, under the assumption of normality, given the 29,190 daily returns considered, 79 (=0.0027·29,190) would be expected to fall outside this interval, 39 below –3.17% and 39 above 3.22%.[6] However, as the exhibit shows, 253 daily returns were observed below –3.17% and 208 above 3.22%, for a total of 461, almost six times as many as would be expected under normality. Again, the data suggests that the daily returns of the Dow clearly depart from normality.

Panel C shows the daily (arithmetic) mean return over the whole sample period, as well as the mean return of the best and worst 10, 20, and 100 days. Relative to a sample-wide mean return of 0.02%, the mean returns of the best 10, 20, and 100 days were 11.10%, 9.37%, and 5.92%, thus being 10.4, 8.8, and 5.5 standard deviations above the mean. The mean return of the worst 10, 20, and 100 days, on the other hand, were –10.46%, –8.73%, and –5.87%, thus being 9.8, 8.2, and 5.5 standard deviations below the mean.

For perspective on the significance of these departures from the mean, consider the following. The lowest of the best 100 daily returns (4.20%) is 3.9 standard deviations above the mean. This implies that we should observe one return of this magnitude or higher every 20,792 trading days, or one every 83 years, or less than two in the 107 years in the sample period considered;[7] and yet 100 such returns were observed. Similarly, the highest of the worst 100 daily returns (–4.28%) is 4 standard deviations below the mean. This implies that we should observe one return of this magnitude or lower every 31,574 trading days, or one every 126 years, or less than one in the 107 years in the sample considered; and yet 100 such returns were observed.

One more interesting perspective. The lowest of the best 10 daily returns (9.19%) is 8.6 standard deviations above the mean. This implies that one return of this magnitude or larger should be observed every 250,890,349,457,896,000 trading days, or one every 1,003,561,397,831,590 years. Assuming that the life of planet Earth is around 4.5 billion years, we

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[4] Under normality, the coefficients of standardized skewness and kurtosis are asymptotically distributed as $N(0, 6/T)$ and $N(0, 24/T)$, where $T$ is the number of observations in the sample. Hence, values of these coefficients outside the range (–1.96, 1.96) indicate, at the 5% level of significance, significant departures from normality.

[5] The fact that the U.S. stock market has a significant degree of negative skewness is well known. However, it is interesting to note that this negative skewness is *fully* driven by Black Monday. If this one observation (out of 29,190) is excluded from the sample, then the distribution of the Dow exhibits significant *positive* skewness.

[6] The number of returns expected outside the interval considered must be equally split between the upper and the lower tails of the distribution. For clarity, all numbers have been rounded to the nearest integer, and for this reason the 39 returns expected on each tail do not add up to 79.

[7] This and all similar calculations assume 250 trading days a year.

should then observe one return of this magnitude or larger every 223,014 lives of our planet; and yet 10 such returns were observed in 107 years.[8]

Finally, consider panel D, which displays the most interesting figures for investors. A $100 investment at the beginning of 1900 turned into $25,746 by the end 2006, and delivered a mean annual compound return of 5.3%.[9] Note that in a sample of 29,190 days, 10 days account for only 0.03% of all the days considered. And yet missing the best 10 days reduced the terminal wealth by 65% to $9,008, and the mean annual compound return one percentage point to 4.3%. Missing the best 20 days reduced the terminal wealth by 83.2% to $4,313, and the mean annual compound return to 3.6%. And missing the best 100 days (0.34% of the days considered) reduced the terminal wealth by a staggering 99.7% to just $83 (less than the initial capital invested), and the mean annual compound return to −0.2%.

It may be worthwhile to briefly reflect about these figures. If, as argued by Browne (2007), finding the 7% of the time that stocks do well is a 'daunting task,' consider the difficulty of finding the 0.03% of the time that determines nearly two thirds of the terminal wealth. Or the difficulty of finding the 0.34% of the time that determines whether or not any wealth is created at all! The odds against successful market timing are simply staggering.

Panel D also considers the impact on terminal wealth of being out of the market during the worst 10, 20, and 100 days. Avoiding the worst 10 days increased the terminal wealth (with respect to a passive investment) by 206% to $78,781, and the mean annual compound return by more than one percentage point to 6.4%. Avoiding the worst 20 days increased the terminal wealth by 531.5% to $162,588, and the mean annual compound return to 7.2%. And avoiding the worst 100 days (0.34% of the days considered) increased the terminal wealth by a staggering 43,396.8% to $11,198,734, and more than doubled the mean annual compound return to 11.5%.

These figures speak for themselves and should help investors notice the odds they are against when trying to successfully time the market. A negligible proportion of days determines a massive creation or destruction of wealth. The odds against successful market timing are just staggering. And, as will be seen in the next section, the evidence from international equity markets points exactly in the same direction.

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[8] If these numbers are hard to assess, consider that given the Dow's distribution of daily returns between inception and Oct/16/1987, Black Monday was an event 21.1 standard deviations above the mean. The probability of observing an event of this magnitude or larger is 3.98E-99. For perspective, note that the probability of observing an event 8.6 standard deviations above the mean is 'only' 3.99E-18.

[9] As indicated in the exhibit, all figures account for capital gains but not for dividends.

## 4. Evidence (II): International Markets

In order to assess whether the odds of successful market timing are better outside the U.S., the evidence from another 15 international equity markets is discussed in this section. Among these international markets the U.S. is included again, but this time through the more comprehensive S&P-500 index. Exhibit 2 shows the 15 markets in the sample, the index representing each market, the number of years and trading days in each market, and the date when the sample for each market begins (ending, in all cases, at the end of 2006). The exhibit also shows the proportion that 10, 20, and 100 days are relative to the period considered for each market.

**Exhibit 2: Data**
This exhibit describes the data, including the markets in the sample; the index representing each market; the numbers of years and days in the sample of each market; and the first day in each market (Start). P10, P20, and P100 are the proportions that 10, 20, and 100 days represent relative to the total number of days in the sample of each market. All data through Dec/31/2006.

| Market | Index | Years | Days | P10 | P20 | P100 | Start |
|---|---|---|---|---|---|---|---|
| Australia | ASX All Ordinaries | 49 | 12,317 | 0.08% | 0.16% | 0.81% | 12/31/1957 |
| Canada | S&P/TSX-300 Composite | 31 | 7,810 | 0.13% | 0.26% | 1.28% | 12/31/1975 |
| France | SBF-250 | 38 | 9,425 | 0.11% | 0.21% | 1.06% | 12/31/1968 |
| Germany | DAX-30 | 47 | 11,786 | 0.08% | 0.17% | 0.85% | 12/31/1959 |
| Hong Kong | Hang Seng Composite | 37 | 9,129 | 0.11% | 0.22% | 1.10% | 12/31/1969 |
| Italy | BCI Global Price | 34 | 8,805 | 0.11% | 0.23% | 1.14% | 12/31/1972 |
| Japan | Nikkei-225 | 52 | 14,377 | 0.07% | 0.14% | 0.70% | 12/31/1954 |
| New Zealand | All Share Capital | 37 | 9,202 | 0.11% | 0.22% | 1.09% | 12/31/1969 |
| Singapore | SES All Share | 41 | 10,396 | 0.10% | 0.19% | 0.96% | 12/31/1965 |
| Spain | Madrid SE General | 35 | 8,017 | 0.12% | 0.25% | 1.25% | 12/30/1971 |
| Switzerland | Switzerland Price | 38 | 9,501 | 0.11% | 0.21% | 1.05% | 12/27/1968 |
| Taiwan | Taiwan SE Cap Weighted | 40 | 11,272 | 0.09% | 0.18% | 0.89% | 01/05/1967 |
| Thailand | SET General | 31 | 7,710 | 0.13% | 0.26% | 1.30% | 12/31/1975 |
| UK | FTSE All Share | 38 | 9,613 | 0.10% | 0.21% | 1.04% | 12/31/1968 |
| USA | S&P-500 Composite | 79 | 20,918 | 0.05% | 0.10% | 0.48% | 12/31/1927 |
| **Average** | | **42** | **10,685** | **0.10%** | **0.20%** | **1.00%** | |

As the exhibit shows, the market with the smallest sample is Thailand (7,710 trading days in 31 years) and the one with the largest the U.S. (20,918 trading days in 79 years), with an average of 10,685 days (42 years) across all 15 markets. The full sample consists of 15 markets, 627 years, and 160,278 trading days.

Exhibit 3 shows summary statistics for the distributions of daily returns of all 15 markets in the sample; these include the minimum and maximum return, arithmetic and geometric mean return, standard deviation, and measures of skewness and kurtosis. Note that all markets had very large daily swings. The highest of the maximum daily returns was in Hong Kong (18.82%) with an average of 10.87% across markets; the lowest of the minimum daily returns was also in Hong Kong (−33.33%) with an average of −14.94% across markets. All markets have a

significant degree of skewness (negative in all cases with the exception of Thailand) and kurtosis. As was the case with the Dow, the departures from normality are clear in all 15 markets.

**Exhibit 3: Summary Statistics**

This exhibit shows, for the indexes and sample periods in Exhibit 2, summary statistics for the series of daily returns, including minimum (Min) and maximum (Max) return; arithmetic (AM) and geometric (GM) mean return; standard deviation (SD); coefficients of skewness (Skw) and kurtosis (Krt); and coefficients of standardized skewness (SSkw) and standardized kurtosis (SKrt).

| Market | Min | Max | AM | GM | SD | Skw | Krt | SSkw | SKrt |
|---|---|---|---|---|---|---|---|---|---|
| Australia | −24.99% | 7.27% | 0.03% | 0.03% | 0.83% | −2.5 | 74.7 | −113.2 | 1693.5 |
| Canada | −11.13% | 9.03% | 0.04% | 0.03% | 0.84% | −0.8 | 12.5 | −27.7 | 225.0 |
| France | −12.93% | 7.68% | 0.04% | 0.03% | 1.05% | −0.4 | 7.9 | −16.3 | 156.3 |
| Germany | −12.81% | 12.75% | 0.03% | 0.02% | 1.18% | −0.1 | 7.7 | −2.5 | 170.9 |
| Hong Kong | −33.33% | 18.82% | 0.07% | 0.05% | 1.87% | −0.6 | 20.4 | −23.7 | 397.9 |
| Italy | −11.20% | 11.56% | 0.04% | 0.03% | 1.29% | −0.2 | 6.6 | −9.3 | 126.0 |
| Japan | −14.90% | 13.24% | 0.03% | 0.03% | 1.05% | −0.2 | 10.0 | −7.5 | 244.0 |
| New Zealand | −13.45% | 9.61% | 0.03% | 0.02% | 0.85% | −0.9 | 20.4 | −35.8 | 398.9 |
| Singapore | −14.90% | 10.76% | 0.03% | 0.03% | 1.05% | −0.3 | 14.8 | −12.0 | 308.2 |
| Spain | −9.28% | 8.36% | 0.04% | 0.04% | 1.09% | −0.1 | 5.2 | −3.5 | 94.6 |
| Switzerland | −10.82% | 6.54% | 0.03% | 0.02% | 0.92% | −0.7 | 9.9 | −27.3 | 196.5 |
| Taiwan | −7.88% | 9.38% | 0.05% | 0.04% | 1.53% | −0.1 | 2.9 | −3.3 | 62.3 |
| Thailand | −14.84% | 12.02% | 0.04% | 0.03% | 1.48% | 0.2 | 9.1 | 8.6 | 162.8 |
| UK | −11.23% | 9.36% | 0.04% | 0.03% | 1.01% | −0.2 | 7.7 | −6.1 | 154.2 |
| USA | −20.47% | 16.60% | 0.03% | 0.02% | 1.13% | −0.1 | 18.4 | −5.6 | 543.3 |
| **Average** | **–14.94%** | **10.87%** | **0.04%** | **0.03%** | **1.14%** | **–0.5** | **15.2** | **–19.0** | **329.0** |

Exhibit 4 shows the number of outliers, defined as those daily returns more than three standard deviations away from the mean. To illustrate the interpretation of the figures in this exhibit, consider the Australian market. The lower end of the interval three standard deviations around the mean is −2.46%; and although 17 returns lower than this magnitude were expected, 73 such returns were observed. The upper end of the same interval is 2.52%; and although 17 returns higher than this magnitude were expected, 53 were observed. That yields a total of 126 observed outliers, almost four times as many as the 33 expected.

As the exhibit shows, not just in Australia but in all 15 markets the number of outliers observed was far larger than the number of those expected. Across all markets, an average of 163 outliers were observed, over five times more than the 29 expected. In all 15 markets, then, assuming normally distributed returns would have led investors to substantially underestimate risk.

Panel A of Exhibit 5 shows, for each market, the mean daily return over the whole sample period as well as the mean return of the best and worst 10, 20, and 100 days. Panel B, in turn, shows the number of standard deviations away from the sample-wide mean these best and worst mean returns are. Note that, on average across all 15 markets, the mean return of the best

100 days was over 100 times larger than the sample-wide mean return. In absolute value, the same was the case regarding the mean return of the worst 100 days.

**Exhibit 4: Outliers – Expected and Observed**

This exhibit shows, for the indexes and sample periods in Exhibit 2, the expected (Exp) and observed (Obs) number of daily returns three standard deviations (SD) below and above the arithmetic mean return (AM); the ratio between the number of these observed and expected returns; and the total number of expected (TE) and observed (TO) returns more than three SDs away from the mean. 'Exp' figures are rounded to the nearest integer.

| | Lower Tail | | | | Upper Tail | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Market | AM–3·SD | Exp | Obs | Ratio | AM+3·SD | Exp | Obs | Ratio | TE | TO | Ratio |
| Australia | –2.46% | 17 | 73 | 4.4 | 2.52% | 17 | 53 | 3.2 | 33 | 126 | 3.8 |
| Canada | –2.48% | 11 | 73 | 6.9 | 2.55% | 11 | 43 | 4.1 | 21 | 116 | 5.5 |
| France | –3.11% | 13 | 79 | 6.2 | 3.19% | 13 | 61 | 4.8 | 25 | 140 | 5.5 |
| Germany | –3.51% | 16 | 85 | 5.3 | 3.57% | 16 | 76 | 4.8 | 32 | 161 | 5.1 |
| Hong Kong | –5.53% | 12 | 77 | 6.2 | 5.67% | 12 | 80 | 6.5 | 25 | 157 | 6.4 |
| Italy | –3.82% | 12 | 71 | 6.0 | 3.91% | 12 | 48 | 4.0 | 24 | 119 | 5.0 |
| Japan | –3.12% | 19 | 132 | 6.8 | 3.19% | 19 | 112 | 5.8 | 39 | 244 | 6.3 |
| New Zealand | –2.51% | 12 | 61 | 4.9 | 2.56% | 12 | 57 | 4.6 | 25 | 118 | 4.7 |
| Singapore | –3.12% | 14 | 90 | 6.4 | 3.18% | 14 | 86 | 6.1 | 28 | 176 | 6.3 |
| Spain | –3.22% | 11 | 52 | 4.8 | 3.31% | 11 | 61 | 5.6 | 22 | 113 | 5.2 |
| Switzerland | –2.74% | 13 | 101 | 7.9 | 2.79% | 13 | 62 | 4.8 | 26 | 163 | 6.4 |
| Taiwan | –4.55% | 15 | 103 | 6.8 | 4.65% | 15 | 81 | 5.3 | 30 | 184 | 6.0 |
| Thailand | –4.40% | 10 | 62 | 6.0 | 4.48% | 10 | 81 | 7.8 | 21 | 143 | 6.9 |
| UK | –3.00% | 13 | 69 | 5.3 | 3.07% | 13 | 60 | 4.6 | 26 | 129 | 5.0 |
| USA | –3.35% | 28 | 180 | 6.4 | 3.40% | 28 | 173 | 6.1 | 56 | 353 | 6.3 |
| **Average** | **–3.39%** | **14** | **87** | **6.0** | **3.47%** | **14** | **76** | **5.2** | **29** | **163** | **5.6** |

Finally, consider Exhibit 6 which, as panel D of Exhibit 1, displays the most interesting figures for investors. Panel A shows the terminal wealth resulting from passively investing 100 units of local currency between the beginning and the end of each market's sample period. It also shows the terminal wealth resulting from not being invested during each market's best and worst 10, 20, and 100 days. Interestingly, with the only exception of Australia, missing the best 100 days (less than 1% of the days considered in the average market) resulted in a terminal wealth lower than the initial capital invested, and, therefore, on negative mean annual compound returns.

Panel B shows the impact on terminal wealth of not being invested during the best and worst 10, 20, and 100 days in each market. On average across all 15 markets, missing the best 10, 20, and 100 days resulted in a reduction in terminal wealth, relative to a passive investment, of 50.8%, 70.7%, and 97.7%. Avoiding the worst 10, 20, and 100 days, in turn, resulted in an increase in terminal wealth of 150.4%, 372.8%, and 26,532.4%, again relative to a passive investment. As the figures for the Dow showed before, then, a very small number of days has a massive impact on long term performance.

**Exhibit 5: Outliers – Averages and Likelihoods**

Panel A of this exhibit shows, for the indexes and sample periods in Exhibit 2, the arithmetic mean return for the whole sample (All); the mean return of the best 10, 20, and 100 days (B10, B20, and B100); and the mean return of the worst 10, 20, and 100 days (W10, W20, and W100). Panel B shows the number of standard deviations away from the arithmetic mean return these last six magnitudes are.

| *Panel A: Averages* | | | | | | | |
|---|---|---|---|---|---|---|---|
| Market | All | B10 | B20 | B100 | W10 | W20 | W100 |
| Australia | 0.03% | 6.03% | 4.97% | 3.06% | –8.65% | –6.60% | –3.53% |
| Canada | 0.04% | 4.88% | 4.23% | 2.78% | –6.76% | –5.30% | –3.24% |
| France | 0.04% | 6.12% | 5.55% | 3.77% | –7.50% | –6.22% | –3.99% |
| Germany | 0.03% | 8.07% | 7.24% | 4.72% | –8.30% | –7.09% | –4.73% |
| Hong Kong | 0.07% | 13.40% | 11.29% | 7.36% | –14.83% | –11.91% | –7.32% |
| Italy | 0.04% | 8.00% | 6.81% | 4.39% | –8.43% | –7.53% | –4.91% |
| Japan | 0.03% | 7.82% | 6.61% | 4.41% | –7.58% | –6.43% | –4.44% |
| New Zealand | 0.03% | 5.44% | 4.71% | 3.04% | –7.84% | –6.00% | –3.29% |
| Singapore | 0.03% | 8.02% | 6.93% | 4.48% | –8.94% | –7.42% | –4.50% |
| Spain | 0.04% | 5.97% | 5.35% | 3.79% | –6.86% | –5.93% | –3.77% |
| Switzerland | 0.03% | 5.56% | 4.96% | 3.31% | –7.08% | –6.01% | –3.88% |
| Taiwan | 0.05% | 7.02% | 6.80% | 5.45% | –6.82% | –6.66% | –5.64% |
| Thailand | 0.04% | 10.22% | 9.20% | 5.93% | –9.05% | –8.08% | –5.40% |
| UK | 0.04% | 6.61% | 5.85% | 3.76% | –6.95% | –5.73% | –3.72% |
| USA | 0.03% | 10.78% | 9.28% | 5.98% | –10.40% | –8.79% | –5.87% |
| **Average** | **0.04%** | **7.60%** | **6.65%** | **4.42%** | **–8.40%** | **–7.05%** | **–4.55%** |
| *Panel B: Likelihoods* | | | | | | | |
| Australia | | 7.2 | 6.0 | 3.7 | 10.5 | 8.0 | 4.3 |
| Canada | | 5.8 | 5.0 | 3.3 | 8.1 | 6.4 | 3.9 |
| France | | 5.8 | 5.2 | 3.6 | 7.2 | 6.0 | 3.8 |
| Germany | | 6.8 | 6.1 | 4.0 | 7.1 | 6.0 | 4.0 |
| Hong Kong | | 7.1 | 6.0 | 3.9 | 8.0 | 6.4 | 4.0 |
| Italy | | 6.2 | 5.3 | 3.4 | 6.6 | 5.9 | 3.8 |
| Japan | | 7.4 | 6.3 | 4.2 | 7.2 | 6.1 | 4.3 |
| New Zealand | | 6.4 | 5.5 | 3.6 | 9.3 | 7.1 | 3.9 |
| Singapore | | 7.6 | 6.6 | 4.2 | 8.5 | 7.1 | 4.3 |
| Spain | | 5.4 | 4.9 | 3.4 | 6.3 | 5.5 | 3.5 |
| Switzerland | | 6.0 | 5.3 | 3.6 | 7.7 | 6.5 | 4.2 |
| Taiwan | | 4.5 | 4.4 | 3.5 | 4.5 | 4.4 | 3.7 |
| Thailand | | 6.9 | 6.2 | 4.0 | 6.1 | 5.5 | 3.7 |
| UK | | 6.5 | 5.7 | 3.7 | 6.9 | 5.7 | 3.7 |
| USA | | 9.6 | 8.2 | 5.3 | 9.3 | 7.8 | 5.2 |
| **Average** | | **6.6** | **5.8** | **3.8** | **7.6** | **6.3** | **4.0** |

Finally, panel C shows, for all markets, the mean annual compound returns of a passive investment, as well as those resulting from not being invested during the best and worst 10, 20, and 100 days. Note that, on average across all 15 markets, missing the best 10 days (less than 0.1% of the days considered in the average market) resulted in a decrease of almost two percentage points in mean annual compound returns to 6.3%. Avoiding the worst 10 days, in turn, resulted in an increase of more than two percentage points in mean annual compound returns to 10.7%. Furthermore, missing the best 100 days turned mean annual compound returns from a positive 8.2% into a negative 2.8%, and avoiding the worst 100 days more than doubled mean annual compound returns from 8.2% to 21.8%.

## Exhibit 6: Terminal Values

Panel A of this exhibit shows, for the indexes and sample periods in Exhibit 2, the terminal value of 100 units of local currency (TV100) invested on the date indicated in the 'Start' column in Exhibit 2 and held passively through Dec/31/2006, not including dividends; such terminal value without being invested during the best 10, 20, and 100 days (–B10, –B20, and –B100); and such terminal value without being invested during the worst 10, 20, and 100 days (–W10, –W20, and –W100). Panel B shows the percent changes of these last six terminal values with respect to TV100. Panel C shows the mean annual compound return in all the scenarios considered.

| *Panel A: Terminal values* | | | | | | | |
|---|---|---|---|---|---|---|---|
| Market | TV100 | –Best10 | –Best20 | –Best100 | –Worst10 | –Worst20 | –Worst100 |
| Australia | 4,254 | 2,370 | 1,613 | 209 | 10,724 | 17,100 | 160,808 |
| Canada | 1,354 | 842 | 592 | 88 | 2,730 | 4,044 | 36,981 |
| France | 2,358 | 1,302 | 801 | 58 | 5,160 | 8,569 | 140,499 |
| Germany | 1,579 | 727 | 391 | 16 | 3,765 | 6,901 | 204,195 |
| Hong Kong | 12,842 | 3,657 | 1,521 | 11 | 66,374 | 170,371 | 28,047,846 |
| Italy | 1,998 | 927 | 537 | 27 | 4,825 | 9,572 | 309,749 |
| Japan | 4,837 | 2,283 | 1,349 | 65 | 10,685 | 18,361 | 460,717 |
| New Zealand | 952 | 561 | 380 | 48 | 2,164 | 3,307 | 27,480 |
| Singapore | 1,907 | 882 | 500 | 24 | 4,877 | 8,955 | 193,337 |
| Spain | 1,890 | 1,058 | 667 | 46 | 3,850 | 6,425 | 88,871 |
| Switzerland | 897 | 522 | 341 | 35 | 1,873 | 3,110 | 47,558 |
| Taiwan | 8,460 | 4,294 | 2,269 | 42 | 17,149 | 33,548 | 2,815,756 |
| Thailand | 809 | 306 | 139 | 3 | 2,092 | 4,376 | 212,206 |
| UK | 1,854 | 979 | 596 | 46 | 3,819 | 6,063 | 82,888 |
| USA | 8,031 | 2,893 | 1,368 | 24 | 24,293 | 51,195 | 3,503,106 |
| *Panel B: Variation in Terminal Values* | | | | | | | |
| Australia | | –44.3% | –62.1% | –95.1% | 152.1% | 302.0% | 3680.3% |
| Canada | | –37.8% | –56.3% | –93.5% | 101.7% | 198.7% | 2631.8% |
| France | | –44.8% | –66.0% | –97.5% | 118.8% | 263.4% | 5857.8% |
| Germany | | –53.9% | –75.3% | –99.0% | 138.4% | 337.1% | 12832.2% |
| Hong Kong | | –71.5% | –88.2% | –99.9% | 416.9% | 1226.7% | 218315.2% |
| Italy | | –53.6% | –73.1% | –98.6% | 141.5% | 379.1% | 15402.7% |
| Japan | | –52.8% | –72.1% | –98.6% | 120.9% | 279.6% | 9423.9% |
| New Zealand | | –41.0% | –60.1% | –95.0% | 127.2% | 247.3% | 2786.4% |
| Singapore | | –53.7% | –73.8% | –98.7% | 155.8% | 369.7% | 10040.3% |
| Spain | | –44.0% | –64.7% | –97.6% | 103.7% | 240.0% | 4601.9% |
| Switzerland | | –41.8% | –62.0% | –96.1% | 108.8% | 246.6% | 5200.1% |
| Taiwan | | –49.2% | –73.2% | –99.5% | 102.7% | 296.6% | 33183.5% |
| Thailand | | –62.2% | –82.8% | –99.7% | 158.7% | 441.1% | 26141.7% |
| UK | | –47.2% | –67.9% | –97.5% | 106.0% | 227.0% | 4369.9% |
| USA | | –64.0% | –83.0% | –99.7% | 202.5% | 537.5% | 43519.0% |
| **Average** | | **–50.8%** | **–70.7%** | **–97.7%** | **150.4%** | **372.8%** | **26532.4%** |
| *Panel C: Mean Annual Compound Returns* | | | | | | | |
| Australia | 8.0% | 6.7% | 5.8% | 1.5% | 10.0% | 11.1% | 16.3% |
| Canada | 8.8% | 7.1% | 5.9% | –0.4% | 11.3% | 12.7% | 21.0% |
| France | 8.7% | 7.0% | 5.6% | –1.4% | 10.9% | 12.4% | 21.0% |
| Germany | 6.0% | 4.3% | 2.9% | –3.8% | 8.0% | 9.4% | 17.6% |
| Hong Kong | 14.0% | 10.2% | 7.6% | –5.8% | 19.2% | 22.3% | 40.4% |
| Italy | 9.2% | 6.8% | 5.1% | –3.7% | 12.1% | 14.4% | 26.7% |
| Japan | 7.7% | 6.2% | 5.1% | –0.8% | 9.4% | 10.5% | 17.6% |
| New Zealand | 6.3% | 4.8% | 3.7% | –2.0% | 8.7% | 9.9% | 16.4% |
| Singapore | 7.5% | 5.5% | 4.0% | –3.4% | 9.9% | 11.6% | 20.3% |
| Spain | 8.8% | 7.0% | 5.6% | –2.2% | 11.0% | 12.6% | 21.4% |
| Switzerland | 5.9% | 4.4% | 3.3% | –2.7% | 8.0% | 9.5% | 17.6% |
| Taiwan | 11.7% | 9.9% | 8.1% | –2.1% | 13.7% | 15.6% | 29.2% |
| Thailand | 7.0% | 3.7% | 1.1% | –11.1% | 10.3% | 13.0% | 28.0% |
| UK | 8.0% | 6.2% | 4.8% | –2.0% | 10.1% | 11.4% | 19.3% |
| USA | 5.7% | 4.4% | 3.4% | –1.8% | 7.2% | 8.2% | 14.2% |
| **Average** | **8.2%** | **6.3%** | **4.8%** | **–2.8%** | **10.7%** | **12.3%** | **21.8%** |

As these figures show, in all cases a very small number of days account for the bulk of returns delivered by equity markets. Investors do not obtain their long term returns smoothly and steadily over time but largely as a result of booms and busts. Being invested on the good days and not invested on the bad days is key to long term performance. But the odds of successfully predicting the days to be in and out of the markets are, unfortunately, close to negligible.

## 5. Evidence (III): International Markets, 1990-2006

The final step of the inquire focuses on the more recent 1990-2006 period. The reason is that the samples in the previous section are, in most cases, too long to be considered a holding period for investors. The 17-year period between 1990 and 2006, however, is long enough to assess long term performance, and also short enough so that it could have been the actual holding period of many investors.

Exhibits A1 and A2 in the appendix summarize the characteristics of the reduced dataset, which covers the same 15 international markets discussed in the previous section but over the shorter 17-year period between 1990 and 2006. As Exhibit A2 shows, in this shorter sample period the minimum and maximum daily returns are considerably smaller (in absolute value) than those in Exhibit 3. And although all 15 markets still display a significant degree of kurtosis, only 9 markets exhibit significant negative skewness; of the other 6, 4 display a significant degree of positive skewness and 2 (one of which is the U.S.) display no significant skewness.

Exhibit 7 shows the number of outliers, again defined as those daily returns more than three standard deviations away from the mean. As was the case in the longer sample periods, in all 15 markets the outliers observed clearly outnumbered those expected, on average by a factor larger than 5. And as was also the case before, assuming normally distributed returns would have led investors to substantially underestimate risk in all markets.

Panel A of Exhibit 8 shows, for all markets, the (arithmetic) mean return over the whole sample period as well as the mean return over the best and worst 10, 20, and 100 days. Note that, on average across all 15 markets, the mean return of the best 100 days and (the absolute value of) the mean return of the worst 100 days both still are over 100 times larger than the sample-wide mean return. The best and worst 10 days are, on average, close to 5 standard deviations away from the mean. Events of this magnitude or larger have a 0.0000287% probability of occurring under the normal distribution, which means that one such event should be observed every 3,488,555 trading days, or one every 13,954 years. Therefore, during this shorter sample period,

daily booms and busts still occur far more often than what the normality assumption would predict.

**Exhibit 7: Outliers – Expected and Observed, 1990-2006**

This exhibit shows, for the indexes and sample periods in Exhibit A1, the expected (Exp) and observed (Obs) number of daily returns three standard deviations (SD) below and above the arithmetic mean return (AM); the ratio between the number of these observed and expected returns; and the total number of expected (TE) and observed (TO) returns more than three SDs away from the mean. 'Exp' figures are rounded to the nearest integer.

| Market | AM–3·SD | Exp | Obs | Ratio | AM+3·SD | Exp | Obs | Ratio | TE | TO | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Australia | –2.29% | 6 | 20 | 3.4 | 2.35% | 6 | 14 | 2.4 | 12 | 34 | 2.9 |
| Canada | –2.55% | 6 | 40 | 6.9 | 2.61% | 6 | 30 | 5.2 | 12 | 70 | 6.1 |
| France | –3.37% | 6 | 41 | 7.1 | 3.44% | 6 | 27 | 4.7 | 12 | 68 | 5.9 |
| Germany | –4.22% | 6 | 40 | 6.9 | 4.30% | 6 | 29 | 5.0 | 12 | 69 | 6.0 |
| Hong Kong | –4.66% | 6 | 29 | 5.1 | 4.78% | 6 | 27 | 4.8 | 11 | 56 | 4.9 |
| Italy | –3.62% | 6 | 36 | 6.1 | 3.69% | 6 | 20 | 3.4 | 12 | 56 | 4.7 |
| Japan | –4.44% | 6 | 20 | 3.5 | 4.42% | 6 | 26 | 4.6 | 11 | 46 | 4.1 |
| New Zealand | –2.63% | 6 | 28 | 4.9 | 2.67% | 6 | 22 | 3.8 | 11 | 50 | 4.4 |
| Singapore | –3.35% | 6 | 34 | 5.9 | 3.39% | 6 | 34 | 5.9 | 12 | 68 | 5.9 |
| Spain | –3.48% | 6 | 30 | 5.3 | 3.57% | 6 | 26 | 4.6 | 11 | 56 | 4.9 |
| Switzerland | –3.02% | 6 | 41 | 7.1 | 3.10% | 6 | 24 | 4.2 | 11 | 65 | 5.7 |
| Taiwan | –5.40% | 6 | 58 | 9.4 | 5.43% | 6 | 43 | 7.0 | 12 | 101 | 8.2 |
| Thailand | –5.35% | 6 | 31 | 5.5 | 5.37% | 6 | 41 | 7.3 | 11 | 72 | 6.4 |
| UK | –2.71% | 6 | 38 | 6.6 | 2.77% | 6 | 30 | 5.2 | 12 | 68 | 5.9 |
| USA | –2.95% | 6 | 28 | 4.8 | 3.02% | 6 | 33 | 5.7 | 12 | 61 | 5.3 |
| **Average** | **–3.60%** | **6** | **34** | **5.9** | **3.66%** | **6** | **28** | **4.9** | **12** | **63** | **5.4** |

Finally, Exhibit 9 displays the terminal values and mean annual compound returns of a passive investment, as well as the impact on terminal wealth of not being invested during the best and worst 10, 20, and 100 days in each market. Panel A shows the terminal wealth resulting from passively investing 100 units of local currency between the beginning of 1990 and the end of 2006. Interestingly, in three markets (Japan, Taiwan, and Thailand) the terminal wealth was lower than the initial capital invested. Missing the best 100 days in Taiwan and Thailand would have resulted in a loss of virtually all the capital invested; in Germany, Hong Kong, Japan, and Singapore the terminal wealth would have been reduced to less than 10% of the initial capital invested.

Panel B shows the impact on terminal wealth resulting from not being invested during the best 10, 20, and 100 days. On average across all 15 markets, missing the best 10, 20, and 100 days resulted in a reduction of 43.3%, 62.3%, and 95.2% in terminal wealth relative to a passive investment. Although these figures are somewhat smaller than the respective figures in Exhibit 6, they still show that a very small number of days have a massive impact on long term performance. Note that, as Exhibit A1 shows, 10, 20, and 100 days are, on average across all 15 markets, only 0.23%, 0.47% and 2.34% of the total number of days considered.

**Exhibit 8: Outliers – Averages and Likelihoods, 1990-2006**

Panel A of this exhibit shows, for the indexes and sample periods in Exhibit A2, the arithmetic mean return for the whole sample (All); the mean return of the best 10, 20, and 100 days (B10, B20, and B100); and the mean return of the worst 10, 20, and 100 days (W10, W20, and W100). Panel B shows the number of standard deviations away from the arithmetic mean return these last six magnitudes are.

| *Panel A: Averages* | | | | | | | |
|---|---|---|---|---|---|---|---|
| Market | All | B10 | B20 | B100 | W10 | W20 | W100 |
| Australia | 0.03% | 3.07% | 2.70% | 1.99% | –4.16% | –3.28% | –2.10% |
| Canada | 0.03% | 4.02% | 3.65% | 2.40% | –5.09% | –4.18% | –2.69% |
| France | 0.03% | 5.62% | 4.87% | 3.25% | –5.25% | –4.68% | –3.34% |
| Germany | 0.04% | 7.02% | 6.33% | 4.09% | –6.79% | –6.07% | –4.24% |
| Hong Kong | 0.06% | 9.34% | 7.58% | 4.69% | –8.85% | –7.48% | –4.63% |
| Italy | 0.03% | 5.31% | 4.63% | 3.31% | –6.13% | –5.40% | –3.59% |
| Japan | –0.01% | 7.50% | 6.32% | 4.18% | –5.90% | –5.38% | –3.90% |
| New Zealand | 0.02% | 4.63% | 3.80% | 2.47% | –5.27% | –4.18% | –2.50% |
| Singapore | 0.02% | 6.56% | 5.44% | 3.37% | –6.19% | –5.34% | –3.35% |
| Spain | 0.05% | 5.26% | 4.68% | 3.30% | –6.01% | –5.19% | –3.39% |
| Switzerland | 0.04% | 5.49% | 4.71% | 2.94% | –5.39% | –4.80% | –3.16% |
| Taiwan | 0.01% | 7.02% | 6.79% | 5.36% | –6.82% | –6.66% | –5.58% |
| Thailand | 0.01% | 10.01% | 9.01% | 5.65% | –8.84% | –7.73% | –5.01% |
| UK | 0.03% | 4.47% | 3.89% | 2.63% | –4.34% | –3.86% | –2.70% |
| USA | 0.04% | 4.84% | 4.33% | 2.93% | –4.81% | –4.07% | –2.78% |
| **Average** | **0.03%** | **6.01%** | **5.25%** | **3.50%** | **–5.99%** | **–5.22%** | **–3.53%** |
| *Panel B: Likelihoods* | | | | | | | |
| Australia | | 3.9 | 3.4 | 2.5 | 5.4 | 4.3 | 2.8 |
| Canada | | 4.6 | 4.2 | 2.8 | 6.0 | 4.9 | 3.2 |
| France | | 4.9 | 4.3 | 2.8 | 4.7 | 4.1 | 3.0 |
| Germany | | 4.9 | 4.4 | 2.9 | 4.8 | 4.3 | 3.0 |
| Hong Kong | | 5.9 | 4.8 | 2.9 | 5.7 | 4.8 | 3.0 |
| Italy | | 4.3 | 3.8 | 2.7 | 5.1 | 4.5 | 3.0 |
| Japan | | 5.1 | 4.3 | 2.8 | 4.0 | 3.6 | 2.6 |
| New Zealand | | 5.2 | 4.3 | 2.8 | 6.0 | 4.8 | 2.8 |
| Singapore | | 5.8 | 4.8 | 3.0 | 5.5 | 4.8 | 3.0 |
| Spain | | 4.4 | 3.9 | 2.8 | 5.2 | 4.5 | 2.9 |
| Switzerland | | 5.3 | 4.6 | 2.8 | 5.3 | 4.8 | 3.1 |
| Taiwan | | 3.9 | 3.8 | 3.0 | 3.8 | 3.7 | 3.1 |
| Thailand | | 5.6 | 5.0 | 3.2 | 5.0 | 4.3 | 2.8 |
| UK | | 4.9 | 4.2 | 2.8 | 4.8 | 4.3 | 3.0 |
| USA | | 4.8 | 4.3 | 2.9 | 4.9 | 4.1 | 2.8 |
| **Average** | | **4.9** | **4.3** | **2.8** | **5.1** | **4.4** | **2.9** |

Avoiding the worst 10, 20, and 100 days, in turn, resulted in an increase in terminal wealth of 87.9%, 204.4%, and 6,268.5%, again relative to a passive investment. And again, although these figures are somewhat smaller than the respective figures in Exhibit 6, they still clearly show the massive impact that very few days can have on long term performance.

Finally, panel C shows, for all markets, the mean annual compound returns of a passive investment, as well as those resulting from not being invested during the best and worst 10, 20, and 100 days. Interestingly, the impact on mean annual compound returns of not being invested during the best and worst days is much larger than was the case in the longer sample periods considered in the previous section.

**Exhibit 9: Terminal Values, 1990-2006**

Panel A of this exhibit shows, for the indexes and sample periods in Exhibit A2, the terminal value of 100 units of local currency (TV100) invested on Dec/31/1989 and held passively through Dec/31/2006, not including dividends; such terminal value without being invested during the best 10, 20, and 100 days (–B10, –B20, and –B100); and such terminal value without being invested during the worst 10, 20, and 100 days (–W10, –W20, and –W100). Panel B shows the percent changes of these last six terminal values with respect to TV100. Panel C shows the mean annual compound return in all the scenarios considered.

| Market | TV100 | –Best10 | –Best20 | –Best100 | –Worst10 | –Worst20 | –Worst100 |
|---|---|---|---|---|---|---|---|
| *Panel A: Terminal values* | | | | | | | |
| Australia | 342 | 253 | 201 | 48 | 524 | 667 | 2,881 |
| Canada | 325 | 219 | 159 | 30 | 549 | 765 | 4,995 |
| France | 294 | 170 | 114 | 12 | 504 | 767 | 8,801 |
| Germany | 368 | 187 | 108 | 7 | 745 | 1,291 | 28,180 |
| Hong Kong | 704 | 290 | 165 | 7 | 1,782 | 3,345 | 81,814 |
| Italy | 291 | 173 | 118 | 11 | 547 | 883 | 11,370 |
| Japan | 44 | 22 | 13 | 1 | 81 | 134 | 2,378 |
| New Zealand | 170 | 108 | 81 | 15 | 294 | 402 | 2,156 |
| Singapore | 202 | 107 | 70 | 7 | 382 | 606 | 6,119 |
| Spain | 524 | 314 | 210 | 20 | 975 | 1,522 | 16,528 |
| Switzerland | 461 | 270 | 184 | 26 | 802 | 1,233 | 11,463 |
| Taiwan | 81 | 41 | 22 | 0 | 165 | 322 | 25,300 |
| Thailand | 77 | 30 | 14 | 0 | 196 | 388 | 13,372 |
| UK | 267 | 173 | 125 | 20 | 417 | 588 | 4,140 |
| USA | 401 | 250 | 172 | 22 | 657 | 922 | 6,790 |
| *Panel B: Variation in Terminal Values* | | | | | | | |
| Australia | | –26.0% | –41.2% | –86.1% | 53.0% | 95.1% | 742.1% |
| Canada | | –32.6% | –51.2% | –90.6% | 68.8% | 135.3% | 1436.1% |
| France | | –42.1% | –61.4% | –95.9% | 71.6% | 160.9% | 2894.3% |
| Germany | | –49.3% | –70.7% | –98.2% | 102.2% | 250.5% | 7547.9% |
| Hong Kong | | –58.8% | –76.6% | –99.0% | 153.2% | 375.3% | 11524.1% |
| Italy | | –40.4% | –59.5% | –96.1% | 88.2% | 204.0% | 3811.8% |
| Japan | | –51.4% | –70.6% | –98.3% | 83.8% | 202.6% | 5273.2% |
| New Zealand | | –36.3% | –52.4% | –91.3% | 72.5% | 136.2% | 1166.6% |
| Singapore | | –47.0% | –65.3% | –96.3% | 89.5% | 200.3% | 2933.0% |
| Spain | | –40.1% | –59.9% | –96.1% | 86.0% | 190.5% | 3054.7% |
| Switzerland | | –41.4% | –60.1% | –94.5% | 74.1% | 167.7% | 2388.9% |
| Taiwan | | –49.2% | –73.1% | –99.5% | 102.7% | 296.6% | 31021.8% |
| Thailand | | –61.5% | –82.2% | –99.6% | 153.1% | 401.8% | 17192.8% |
| UK | | –35.4% | –53.3% | –92.5% | 55.9% | 119.8% | 1448.4% |
| USA | | –37.7% | –57.1% | –94.4% | 63.8% | 129.7% | 1591.8% |
| **Average** | | **–43.3%** | **–62.3%** | **–95.2%** | **87.9%** | **204.4%** | **6268.5%** |
| *Panel C: Mean Annual Compound Returns* | | | | | | | |
| Australia | 7.5% | 5.6% | 4.2% | –4.3% | 10.2% | 11.8% | 21.9% |
| Canada | 7.2% | 4.7% | 2.8% | –6.7% | 10.5% | 12.7% | 25.9% |
| France | 6.5% | 3.2% | 0.8% | –11.7% | 10.0% | 12.7% | 30.1% |
| Germany | 8.0% | 3.7% | 0.5% | –14.7% | 12.5% | 16.2% | 39.4% |
| Hong Kong | 12.2% | 6.5% | 3.0% | –14.3% | 18.5% | 22.9% | 48.4% |
| Italy | 6.5% | 3.3% | 1.0% | –12.0% | 10.5% | 13.7% | 32.1% |
| Japan | –4.7% | –8.6% | –11.3% | –25.1% | –1.2% | 1.7% | 20.5% |
| New Zealand | 3.2% | 0.5% | –1.2% | –10.6% | 6.5% | 8.5% | 19.8% |
| Singapore | 4.2% | 0.4% | –2.1% | –14.2% | 8.2% | 11.2% | 27.4% |
| Spain | 10.2% | 7.0% | 4.5% | –8.9% | 14.3% | 17.4% | 35.0% |
| Switzerland | 9.4% | 6.0% | 3.6% | –7.7% | 13.0% | 15.9% | 32.2% |
| Taiwan | –1.2% | –5.1% | –8.6% | –27.3% | 3.0% | 7.1% | 38.5% |
| Thailand | –1.5% | –6.9% | –11.0% | –28.7% | 4.0% | 8.3% | 33.4% |
| UK | 6.0% | 3.3% | 1.3% | –9.0% | 8.8% | 11.0% | 24.5% |
| USA | 8.5% | 5.5% | 3.2% | –8.4% | 11.7% | 14.0% | 28.2% |
| **Average** | **5.5%** | **1.9%** | **–0.6%** | **–13.6%** | **9.4%** | **12.3%** | **30.5%** |

Across all 15 markets, and relative to a passive investment, missing the best 10 days reduced mean annual compound returns by over three percentage points to 1.9%; missing the best 20 days resulted in negative mean annual compound returns in 5 markets, and on average across all markets; and missing the best 100 days (2.34% of the days considered in the average market) resulted in negative mean annual compound returns in *all* markets. Avoiding the worst 10 days, in turn, increased mean annual compound returns by almost four percentage points to 9.4%; avoiding the worst 20 days resulted in more than doubling mean annual compound returns to 12.3%; and avoiding the worst 100 days resulted in mean annual compound returns of 30.5%, over five times higher than those of a passive investment.

The evidence from this more recent period, then, confirms and strengthens the results and implications of the previous two sections: A very small number of days account for the bulk of stock market returns and, for this reason, investors are very unlikely to successfully predict the right times to be in and out of the market.

## 6. An Assessment

> "*Investors that design portfolios using standard statistical measures may underestimate risk … Many of the most spectacular failures in the hedge fund world have been the direct result of fat-tail events. Investors need to take these events into consideration when constructing portfolios.*" Mauboussin (2006).

The assumption of normally-distributed returns is pervasive in finance; it is widely used and abused, implicitly or explicitly, by both academics and practitioners. And yet the evidence of the last 40+ years clearly disputes the plausibility of this assumption, particularly as far as daily returns are concerned. Unfortunately, many widely-used measures of risk stem from the normality assumption and basically exclude the possibility of black swans. The fractal framework suggested by Mandelbrot and Hudson (2005) does not solve the problems created by black swans, but at least it makes them conceivable.

Large daily swings that have a significant impact on long term performance, unexpected ex-ante though seemingly predictable ex-post, occur far more often than what the normality assumption would lead investors to believe. Black swans do exist. In fact, investors stumble upon them far more often than they expect, and the impact on their portfolios is far larger than they usually think.

The evidence discussed in this article, based on 15 international equity markets and over 160,000 daily returns, clearly shows that black swans have a massive impact on long term performance. On average across all 15 markets, missing the best 10 days resulted in portfolios 50.8% less valuable than a passive investment; and avoiding the worst 10 days resulted in

portfolios 150.4% more valuable than a passive investment. For the average market, then, less than 0.1% of the days considered swung long term returns by more than 50% above or below those of a passive investment.

Two recommendations seem to follow from these results, both based on the fact that black swans are largely unpredictable and have a massive impact on long term performance. Taleb (2007) recommends to adjust to the existence of black swans rather than trying to predict them. It is possible to have an idea of the consequences of an event even if its probability of occurring is largely unknown (think of an earthquake in San Francisco or a flood in New Orleans), and it is on these potential consequences that investors should focus. Hence, broad diversification would mitigate exposure to negative black swans while preserving some exposure to positive black swans.

Second, black swans render market timing a goose chase. Attempting to predict the negligible proportion of days that determines an enormous creation or destruction of wealth seems to be a losing proposition. Of the countless strategies that academics and practitioners have devised to generate alpha, market timing seems to be one very unlikely to succeed. Much like going to Vegas, market timing may be an entertaining pastime, but not a good way to make money.

# Appendix

### Exhibit A1: Data, 1990-2006

This exhibit describes the data, including the markets in the sample; the index representing each market; the numbers of years and days in the sample of each market; and the first day in each market (Start). P10, P20, and P100 are the proportions that 10, 20, and 100 days represent relative to the total number of days in the sample of each market. All data through Dec/31/2006.

| Market | Index | Years | Days | P10 | P20 | P100 | Start |
|---|---|---|---|---|---|---|---|
| Australia | ASX All Ordinaries | 17 | 4,295 | 0.23% | 0.47% | 2.33% | 12/31/1989 |
| Canada | S&P/TSX-300 Composite | 17 | 4,279 | 0.23% | 0.47% | 2.34% | 12/31/1989 |
| France | SBF-250 | 17 | 4,283 | 0.23% | 0.47% | 2.33% | 12/31/1989 |
| Germany | DAX-30 | 17 | 4,281 | 0.23% | 0.47% | 2.34% | 12/31/1989 |
| Hong Kong | Hang Seng Composite | 17 | 4,207 | 0.24% | 0.48% | 2.38% | 12/31/1989 |
| Italy | BCI Global Price | 17 | 4,371 | 0.23% | 0.46% | 2.29% | 12/31/1989 |
| Japan | Nikkei-225 | 17 | 4,187 | 0.24% | 0.48% | 2.39% | 12/31/1989 |
| New Zealand | All Share Capital | 17 | 4,254 | 0.24% | 0.47% | 2.35% | 12/31/1989 |
| Singapore | SES All Share | 17 | 4,264 | 0.23% | 0.47% | 2.35% | 12/31/1989 |
| Spain | Madrid SE General | 17 | 4,229 | 0.24% | 0.47% | 2.36% | 12/31/1989 |
| Switzerland | Switzerland Price | 17 | 4,259 | 0.23% | 0.47% | 2.35% | 12/31/1989 |
| Taiwan | Taiwan SE Cap Weighted | 17 | 4,578 | 0.22% | 0.44% | 2.18% | 12/31/1989 |
| Thailand | SET General | 17 | 4,171 | 0.24% | 0.48% | 2.40% | 12/31/1989 |
| UK | FTSE All Share | 17 | 4,296 | 0.23% | 0.47% | 2.33% | 12/31/1989 |
| USA | S&P-500 Composite | 17 | 4,285 | 0.23% | 0.47% | 2.33% | 12/31/1989 |
| **Average** | | **17** | **4,283** | **0.23%** | **0.47%** | **2.34%** | |

### Exhibit A2: Summary Statistics, 1990-2006

This exhibit shows, for the indexes and sample periods in Exhibit A1, summary statistics for the series of daily returns, including minimum (Min) and maximum (Max) return; arithmetic (AM) and geometric (GM) mean return; standard deviation (SD); coefficients of skewness (Skw) and kurtosis (Krt); and coefficients of standardized skewness (SSkw) and standardized kurtosis (SKrt).

| Market | Min | Max | AM | GM | SD | Skw | Krt | SSkw | SKrt |
|---|---|---|---|---|---|---|---|---|---|
| Australia | –7.18% | 6.25% | 0.03% | 0.03% | 0.77% | –0.3 | 4.6 | –9.1 | 61.5 |
| Canada | –8.12% | 4.79% | 0.03% | 0.03% | 0.86% | –0.6 | 6.7 | –15.4 | 88.9 |
| France | –7.18% | 6.47% | 0.03% | 0.03% | 1.14% | –0.1 | 3.4 | –2.8 | 46.0 |
| Germany | –9.40% | 7.85% | 0.04% | 0.03% | 1.42% | –0.1 | 3.9 | –3.5 | 52.6 |
| Hong Kong | –13.70% | 18.82% | 0.06% | 0.05% | 1.57% | 0.2 | 11.3 | 6.5 | 149.3 |
| Italy | –8.12% | 6.58% | 0.03% | 0.02% | 1.22% | –0.3 | 3.2 | –7.6 | 42.6 |
| Japan | –6.98% | 13.24% | –0.01% | –0.02% | 1.48% | 0.3 | 3.5 | 7.4 | 45.9 |
| New Zealand | –12.10% | 9.61% | 0.02% | 0.01% | 0.88% | –0.4 | 14.6 | –11.3 | 193.8 |
| Singapore | –8.01% | 9.16% | 0.02% | 0.02% | 1.12% | 0.1 | 7.1 | 2.3 | 94.6 |
| Spain | –8.25% | 5.89% | 0.05% | 0.04% | 1.18% | –0.2 | 3.4 | –6.0 | 45.3 |
| Switzerland | –6.67% | 6.54% | 0.04% | 0.04% | 1.02% | –0.2 | 5.2 | –6.1 | 69.4 |
| Taiwan | –7.88% | 9.38% | 0.01% | 0.00% | 1.81% | –0.1 | 2.5 | –1.6 | 35.2 |
| Thailand | –14.84% | 12.02% | 0.01% | –0.01% | 1.79% | 0.3 | 6.0 | 8.1 | 78.6 |
| UK | –5.21% | 5.86% | 0.03% | 0.02% | 0.91% | –0.1 | 3.5 | –2.3 | 47.4 |
| USA | –6.87% | 5.73% | 0.04% | 0.03% | 1.00% | 0.0 | 3.8 | –0.4 | 51.0 |
| **Average** | **–8.70%** | **8.55%** | **0.03%** | **0.02%** | **1.21%** | **–0.1** | **5.5** | **–2.8** | **73.5** |

## References

Aparicio, Felipe, and Javier Estrada (2001). "Empirical Distributions of Stock Returns: European Securities Markets, 1990-95." European Journal of Finance, 7, 1-21.

Browne, Christopher (2007). *The Little Book of Value Investing*. Wiley.

Fama, Eugene (1965). "The Behavior of Stock Market Prices." Journal of Business, 38, 34-105.

Haugen, Robert (1999). *Beast on Wall Street. How Stock Volatility Devours Our Wealth*. Prentice Hall.

Jansen, Dennis, and Casper de Vries (1991). "On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective." Review of Economics and Statistics, 73, 18-24.

Longin, Francois (2005). "The Choice of the Distribution of Asset Returns: How Extreme Value Theory Can Help?" Journal of Banking and Finance, 29, 1017-1035.

Mandelbrot, Benoit (1963). "The Variation of Certain Speculative Prices." Journal of Business, 36, 394-419.

Mandelbrot, Benoit, and Richard Hudson (2005). *The (Mis)Behavior of Markets. A Fractal View of Risk, Ruin And Reward*. Profile Books.

Mauboussin, Michael (2006). *More Than You Know. Finding Financial Wisdom in Unconventional Places*. Columbia University Press.

Peters, Edgar (1991). *Chaos and Order in the Capital Markets. A New View of Cycles, Prices, and Market Volatility*. John Wiley and Sons.

Taleb, Nassim (2007). *The Black Swan. The Impact of the Highly Improbable*. Random House.