

A REEXAMINATION OF CREDIT SPREAD COMPONENTS

Lawrence Kryzanowski¹

Wassim Dbouk²

Concordia University

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¹Ned Goodman Chair in Investment Finance. Finance Department, John Molson School of Business, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec, Canada, H3G 1M8. Telephone: (514) 848-2424, ext. 2782; e-mail: lkryzan@vax2.concordia.ca.

²Finance Department, John Molson School of Business, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec, Canada, H3G 1M8. E-mail: Wassim Dbouk<wdbouk@jmsb.concordia.ca>.

Comments are welcomed.

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Abstract

Using a modified version of the methodology used in Elton et al. (2001), this paper reexamines how default, taxes and systematic risk measures influence corporate credit spreads for investment grade corporate bonds for the 1987-1996 time period. The methodological improvements not only change the estimates for the default and tax components of credit spreads materially but the factors from the Fama and French three-factor model no longer help to explain the remaining variation in credit spreads. In contrast, a good portion of the variation in the remaining (unexplained) spread is explained by measures of aggregate bond liquidity.

A REEXAMINATION OF CREDIT SPREAD COMPONENTS

1. INTRODUCTION

Credit spreads are of increasing interest in the academic literature and have long been of interest in corporate practice. While credit spreads are often generally perceived as being compensation for credit risk, the time-series behavior of credit spreads is not yet well understood. Elton, Gruber, Agrawal and Mann (2001) (henceforth EGAM) provide estimates of the size of each factor-related component of the credit spread for investment-grade corporate bond portfolios (namely, the default spread, tax spread, and risk premium).

Our analysis finds that EGAM did not address three potentially important issues when making their estimations. In short, EGAM's default spread depends on the one-year transition matrix published by Moody's. Nickell et al. (2001) show that transition matrices depend on the country of domicile, the industry, and the phase in the business cycle. As expected, business-cycle-conditioned, sector-specific transition matrices differ significantly from the one used by EGAM. A second shortcoming is the absence of federal taxes and amortization effects and other important complexities in the tax system on EGAM's tax computations. Wang et al. (2005) show that these factors are important and could change tax measurements significantly. Finally, although EGAM note that liquidity may affect credit spreads and the literature has long alluded to the existence of a liquidity component in credit spreads, estimates of the impact of the liquidity component on credit spreads is absent in the EGAM study.

Given these shortcomings, the primary objective of this paper is to estimate the default spread in the light of the findings of Nickell et al. (2001), to reestimate the tax effect by more carefully modeling the intricacies of the actual tax code, and to examine the portion of the spreads attributable to systematic risk and aggregate liquidity.

This paper makes three important contributions to the literature. The first contribution consists of better estimates of the components of credit spreads than the ones previously reported in the literature, since our estimates are based on the recent findings regarding the estimation of transition matrices and the tax effect. The second contribution is to show that the use of an improved estimation methodology leads to different estimates for the various components of credit spreads, and that the macro-factors effect reported by EGAM no longer plays any role in credit-spread determination. The third contribution is to show that aggregate market liquidity plays an important role in the determination of credit spreads. Thus, the significant relations found between stock returns and aggregate liquidity by Chordia et al. (2001), Amihud (2002), amongst others, also applies to bond credit spreads.

This paper is organized as follows. The literature on credit-spread decomposition is presented in the next section. The databases and data selection procedures used herein are described in section three. Methods used to compute the credit spreads are detailed in section four. The decomposition of credit spreads into default spreads, tax spreads, and risk premia are reported and analyzed in section five. The findings on the liquidity credit-spread effect are reported and analyzed in section six. Section seven concludes the paper.

2. LITERATURE REVIEW

The existing literature on the determinants of credit spreads is limited. EGAM (2001) examine the spreads in the rates between corporate and government bonds by decomposing the credit spread into three components; namely, default risk, taxes and a residual. EGAM find that default risk accounts for only a small portion of credit spreads, which is consistent with most credit-spread studies. Collin-Dufresne et al. (2001) find that factors associated with default risk explain only about 25% of the changes in credit spreads. Huang and Huang (2003) find

confirming results using a structural model estimated on the same datasets as EGAM. However, using a continuous time all-sectors transition matrix and a methodology similar to that of EGAM,¹ Dionne et al. (2004) suggest that these studies may have underestimated the portion of corporate spreads explained by default risk since their estimates of the proportional contribution of default spreads are as high as 80% of the estimated spreads.

EGAM (2001) also examine how much of the time-series variation in the residual spread can be explained by systematic risk factors. They find that the Fama and French (1993) factors explain substantial variations in credit-spread changes. Collin-Dufresne et al. (2001) report that a dominant but unidentified systematic factor accounts for about 75% of the variation in spreads. They also find that, while aggregate market factors (such as the level and volatility of interest rates, the volatility of the equity market, and the Fama and French (1993) factors) are more important than issuer-specific characteristics in determining credit spread changes, these factors provide limited additional explanatory power over the default risk factors.

Leland and Toft (1996) claim that the Treasury yield influences not only the discount rate but also directly influences the value of the underlying asset. Thus, the value of the firm decreases and the probability of default correspondingly increases as the Treasury yield increases. In turn, this implies a positive relation between credit spreads and the level of Treasury yields. Duffee (1998) finds a significant, although weaker, negative relationship between changes in credit spreads and Treasury interest rates, which he claims is consistent with the contingent claims approach of Merton (1974) where the firm is valued in an option-theoretic framework. In the Merton model, an increase in the level of the Treasury rate increases the value of the firm. In turn, this should lower the probability of default by moving the price farther away from the exercise price. Morris et al. (2002) show that the relation is positive (and not negative) in the

¹ Dionne et al use a theoretical 10-year, zero-coupon bond instead of the coupon-paying bond used by EGAM.

long run. Using a reduced-form model to decompose spreads into taxes, liquidity risk, common factor risks, default event risk, and firm-specific factor risks, Driessen (2002) finds that the default jump risk premium explains a significant portion of corporate bond returns.

To summarize, the literature on credit spreads suggests that factors such as the level of the Treasury interest rate, systematic risk, firm-specific risk, liquidity, and taxation play an important role in determining credit spreads.

3. SAMPLE AND DATA

To maintain comparability with the findings of EGAM, our bond data are extracted from the Lehman Brothers Fixed Income Database distributed by Warga (1998). This database contains monthly clean prices and accrued interest on all investment grade corporate and government bonds. In addition, the database contains descriptive data on bonds including coupons, maturities, principals, ratings, and callability. Our sample includes 10 years of monthly data from 1987 through 1996.² All bonds with embedded options, such as callable, puttable, convertible, and sinking fund bonds, are eliminated. Similarly, all corporate floating-rate debt and bonds with an odd frequency of coupon payment (i.e., other than semi-annual) are eliminated from the sample.³ Furthermore, all bonds not included in the Lehman Brothers bond indexes are eliminated because, as EGAM report, much less care occurs in preparing the data for these non-index bonds. This leads to the elimination of, for example, all bonds with a maturity of less than one year. A \$5 pricing error filter is used also to eliminate bonds where the price data are problematic.

² The results for the 1987-1997 period are not materially different from those for the 1987-1996 period.

³ While EGAM eliminate government flower bonds and inflation-indexed government bonds, these bonds could not be identified even when using the FISD database. Since no flower bonds are issued after March 3, 1971 and since no flower bonds have maturities after 1998, we eliminate the few treasury bonds that were issued prior to 1971 and were due to mature before 1998. Regarding inflation-indexed government bonds, we eliminate the variable rate bonds from our sample. We assume that these inflation-indexed bonds are included in the elimination process although there is no information about which bonds are inflation-indexed bonds.

Also, following Elton et al (2001), bonds maturing after 10 years are eliminated. Since Kryzanowski and Xu (1997) show that the yields from both extremes of the one-to-thirty-year term structure do not exhibit clear pairwise cointegration, we find it also more appropriate to eliminate the bonds maturing after 10 years (as in EGAM) since these very long maturities are driven by different factors than those driving the short-term spot rates.

Only the prices based on dealer quotes are extracted. All matrix-based prices are eliminated from the sample since matrix prices might not reflect fully the economic influences in the bond markets. Since we are unable to identify the frequency of payments and the nature of coupons (fixed or variable) from the Warga (1998) database, we rely on the descriptive statistics from *The Fixed Income Securities Database (FISD)* to identify various bond characteristics. *FISD* contains all insurance company daily buys and sells of US corporate bonds for the 1995-1999 period, and reports more extensive bond details than those provided by Warga (1998).

Since the number of buy and sell prices in the FISD are limited, the term structures of the credit spreads could be extracted for only a few bond categories (such as the Aa-rated industrial bonds) from the FISD database. Consequently, the FISD prices could not be used to derive the components of the credit spreads.

Our study is focused on the industrial sector since our methodology, as is explained later, requires an estimation of the after-default and after-tax term structures as well as the before-default and before-tax term structures. By focusing on this sector, computation time is decreased significantly. It is our belief that computational time constraints induced EGAM to take a short cut, which is shown later as leading to a number of estimation drawbacks.

The final sample consists of 59,463 bond prices from which 47,000 bond prices are corporate and 12,463 are Treasury. Of this total, 14,754 are for Aa-rated bonds, 18,031 bond prices are for A-rated bonds and 14,215 are for Baa-rated bonds.

4. SPREAD MEASURES

4.1 Basic Results

Most previous work has considered credit spreads as being the conventional difference between the yields to maturity on corporate and Treasury bonds with similar maturities. Due to the effect of the coupon level on yields-to-maturity and measures of risk, EGAM note that credit spreads should be considered as the difference between the yield to maturity on a zero-coupon corporate bond (corporate spot rate) and the yield to maturity on a zero-coupon government bond of the same maturity (government spot rate). Extracting the yields to maturity from coupon-paying bonds results in a term structure being extracted from bonds with different durations and convexities.

The Nelson-Siegel (1987) procedure, which is briefly described in Appendix A and is used by many central banks, is used herein to estimate the zero-coupon spot rates from coupon carrying bonds. This procedure is chosen because it has enough flexibility to reflect the patterns of the observed market data, is relatively robust against disturbances from individual observations, and is applicable with a small number of observations.

The Nelson and Siegel approach uses the following equation:

$$r(t) = \beta_0 + \beta_1 \left[\frac{1 - \exp(-t/\tau_1)}{t/\tau_1} \right] + \beta_2 \left[\frac{1 - \exp(-t/\tau_1)}{t/\tau_1} - \exp(-t/\tau_1) \right] \quad (1)$$

where r is the estimated spot rate with maturity t . The four parameters, β_0 , β_1 , β_2 , and τ_1 , need to be estimated in order to estimate the spot rates.

The corporate spot rate curve is estimated for each of the three bond-rating categories of Aa, A and Baa for the industrial sector.⁴ The estimated spot rates with maturities from 1 to 10 years are obtained by minimizing the sum of squared pricing errors using a four-step estimation procedure. In the first step, the TOMLAB (OQNLP solver) software, which starts with different sets of initial values and returns the global minima, is used.⁵ The reason is that, since the optimization toolbox in Matlab provides spot rate estimates that are very sensitive to the selected vector of starting parameters (β_0 , β_1 , β_2 , and τ_1), there is a high probability that the solution converges to a local and not global minimum. The second step is to determine the discount factors corresponding to the coupon and face value payment dates using these starting parameters. The third step is to calculate the theoretical dirty prices of the bonds by discounting the bond cash flows to time t (the quote dates). Numerical optimization procedures are used to re-estimate the set of parameters that minimizes the sum of squared price errors between the observed dirty prices and the theoretical ones. The fourth and last step is to use the estimated set of parameters (β_0 , β_1 , β_2 , and τ_1) to determine the spot rate function by plugging these estimated parameters into Equation (1) and assigning maturities ranging from 1 to 10 years. The estimated spot rates are the annual continuously compounded zero-coupon spot rates.

The resulting spot rate estimates, which are summarized in panel A of table 1, are consistent with the theory. All the empirical bond-spread curve estimates are positive and increasing as the rating class deteriorates. This strongly suggests that ratings are indeed linked to credit quality. Furthermore, the credit spreads are upward-sloping exhibiting higher credit spreads with lower ratings, and higher credit spreads with longer maturities.

⁴ Technically, the corporate instantaneous forward rate curve is derived first, which gives an estimate of the four parameters β_0 , β_1 , β_2 , and τ_1 . After that the corporate spot rate curve is estimated.

⁵ The TOMLAB Optimization Environment is a powerful optimization platform for solving applied optimization problems in Matlab. The solution is independent of the starting values. The starting values are calculated from a scatter search algorithm. TOMLAB provides a multi-start algorithm designed to find global optima.

[Please place table 1 about here.]

Based on the root mean squared errors (RMSEs) that are reported in panel B of table 1, our estimates produce acceptable average RMSEs that range from \$ 0.362 per \$100 for Treasuries to \$1.570 per \$100 for Baa bonds. Our average RMSEs are slightly higher than those reported by EGAM but this could be attributed to the apparent elimination of fewer outliers from our sample (947 industrial bond prices herein whereas 2,710 industrial and financial bond prices in EGAM).

5. SPREAD DECOMPOSITION

5.1 Default Spread Estimates Based on the Unmodified EGAM Approach

Although the expected loss on corporate bonds due to default is an obvious component of credit spreads, most of the previous studies find that the default premium accounts for a surprisingly small fraction of credit spreads. The findings reported in this section support these previous findings.

The EGAM methodology is used to estimate the proportional representation of the default spread. Under risk neutrality with the tax effect ignored, the difference between the corporate and forward rates is given by:

$$e^{-(f_{t+1}^C - f_{t+1}^G)} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C} \quad (2)$$

where f_{t+1}^C and f_{t+1}^G are the forward rates as of time t from t to $t+1$ for corporate and government bonds, respectively; P_{t+1} is the conditional probability of default between t and $t+1$ given no bankruptcy at t ; a is the recovery rate; V_{t+1T} is the value of a T -period bond at $t+1$ given no bankruptcy in earlier periods; and C is the coupon rate.

To calculate the risk-neutral spread in forward rates, the marginal default probability, the recovery rate, and the coupon rate need to be estimated. To calculate the conditional probability of default, the one-year transition matrix from Moody's (see table 2) is used to calculate the default probabilities of year 1 by simply taking the default probabilities indicated in the last column and ascribing them to bonds with corresponding credit ratings. For example, a Baa-rated bond is assigned a 0.103% probability of default within one year using this approach. A similar approach is used for longer-term unconditional default probabilities. For example, the matrix is multiplied by itself $(n-1)$ times to obtain the n -year unconditional default probabilities, where the desired default probabilities are given in the default (last) column of that matrix. Similarly, the conditional default probabilities for year $t+1$ are computed as the difference between the unconditional default probabilities for years $t+1$ and t , all divided by the probability of not defaulting in year t .⁶

[Please place table 2 about here.]

The Altman and Kishore (1998) estimates of recovery rates by rating class that are reported in panel C of table 2 are used herein. These estimates are based on actual recovery rates observed in practice based on an examination of 696 defaulted bond issues over the period 1975-1995. As in EGAM, the coupon rate that makes the value of a 10-year bond approximately equal to the par value of the bond in all periods is used to estimate the default spread.

The forward rates are obtained assuming risk neutrality and zero taxes using equation 2 along with the conditional default probabilities from table 3, recovery rate estimates from table 2 and coupon rates estimated as explained earlier. Forward rates are then used to compute the spot rate

⁶ Bayes' theorem is used to obtain the conditional probability of default (that is, the probability of default between time t and time $t+1$), which is given by $[s(t+1) - s(t)]/s(t)$ where s or the probability of surviving the previous period is calculated as 1- probability of default. The probability of default is obtained from the transition matrix. Specifically, the probability of default after n years is calculated by multiplying the matrix by itself n times and extracting the relevant numbers from the default column that correspond to the investment grade rating of the bond.

spreads.⁷ As reported previously in the literature, the default spreads using the EGAM methodology for our sample that are reported in table 4 account for only a small percentage of credit spreads. For example, the default spreads for bonds maturing after 10 years are only 0.014%, 0.05% and 0.35% for Aa-, A- and Baa-rated bonds, respectively. This small increase in the default spread for Baa- versus Aa-rated bonds is attributed mainly to a higher default probability and a lower recovery rate. For instance, the default probability and the recovery rate are 0.146 % and 59.59%, respectively, for an Aa-rated bond, compared to 1.264% and 49.42%, respectively, for a Baa-rated bond.

[Please place tables 3 and 4 about here.]

5.2 Default Spread Estimates Based on an Improved Estimation Methodology

In this section, we outline how the EGAM methodology used in the previous section to estimate default probabilities can be improved. This includes the use of sector-specific, conditional default probabilities that are dependent on the phase of the business cycle, and the computation of the default spreads based on the after-default corporate spot rates instead of a theoretical 10-year, par value bond.

5.2.1 Transition Matrix for the Industrial Sector

In this section, we deal with our first concern with the EGAM methodology; that is, with the use of a theoretical par value bond with an estimated coupon rate that does not disturb the par value property. We argue that estimating the default spread as the difference between the spot rate curves computed in section 4.1 and the after-default term structures computed from the data should be more accurate since this spread is based on the actual data and not on a theoretical bond.

⁷ The relationship between the n period forward rate at time t , $r_{t,n}^f$, and the spot rates is $Exp[r_{t,n}^f] = (Exp[(t+n)r_{t+n}] / Exp[tr_t])^{1/n}$.

Our initial sample for building these transition matrices consists of all ratings in Moody's Default Risk Service (DRS) database for the industrial, US-based, senior and unsecured corporate bonds from the 1970-1998 period.⁸ As is the common approach in the literature (Carty, 1997; Nickell et al., 2000; among others), withdrawn ratings are removed from the sample. The final sample consists of 23,645 yearly bond ratings for 2,144 obligors.⁹ To estimate the transition matrix probabilities, the cohort approach is used after combining the C, Ca and Caa ratings due to the paucity of observations in the C and Ca categories and given that our main concern is to study the spread of investment grade bonds.¹⁰

Our industrial-sector transition results without reflecting business cycle effects (panel B of table 2) are quite different from the all-sectors estimates (panel A of table 2) reported by Carty and Fons (1994) and used in the EGAM study. Our transition results show higher default probabilities for certain categories (Baa and BB) and lower probabilities for other categories (B and Caa). Except for the Aa category, our results exhibit a greater tendency to remain in their initial rating category over the next year.

To assess the impact of using the industrial versus all-sectors transition matrix, we compare the evolution of these conditional default probabilities over the 10-year period for the Aa-, A-, and Baa-rated bonds. Based on table 3, conditional default probabilities for the industrial sector are significantly lower than those for the all-sectors. Consequently, we would expect to obtain lower default spreads for the industrial sector than those reported by EGAM. As expected, the

⁸ This database not only contains detailed information about bonds rated by Moody's that defaulted but it also contains the historical ratings for all bonds rated by Moody's along with other descriptors such as industry and country of domicile of the borrower. This sample period was chosen because the database has complete data for this period. On the other hand, most of the studies in the literature including the rating agency estimates are based on only senior unsecured bonds. For details about Moody's approach, refer to Carty (1997).

⁹ The 7,632 ratings obtained from the ratings master file are allocated to yearly ratings. For example, if a rating is from 1/1/1994 until 12/31/1996, then this rating is used for the years from 1/1/1994 until 12/31/1994, 1/1/1995 until 12/31/1995, and 1/1/1996 until 12/31/1996.

¹⁰ The empirical transition matrix has probabilities $P_{ij} = N_{ij}/M_i$ where N_{ij} is the number of times that the credit rating went from i to j in one year, and M_i is the number of times the credit rating started at i .

probabilities reported in table 4 are substantially lower using our methodology instead of the unaltered methodology of EGAM. For example, the default spreads using the industrial versus all-sector transition matrix for bonds maturing after 10 years in our sample are lower by 44%, 47% and 37% for Aa-, A- and Baa-rated bonds, respectively. Similarly, the default spreads using the industrial transition matrix and our sample are lower than those reported by EGAM by 71%, 66% and 37% for the Aa-, A- and Baa-rated bonds, respectively.

5.2.2 The Business Cycle Effect

In this section, we deal with our second concern with the EGAM methodology; that is, with the use of default probabilities derived from Moody's one-year transition matrix since this matrix does not capture the relationship of rating transitions with the phase of the business cycle, as found by Nickell et al. (2000). Thus, we build our own one-year transition matrices to capture the business cycle effect.

The first step in this adjustment procedure is to use the data published by the Bureau of Economic Analysis (BEA) on real GDP growth to identify the thresholds that differentiate between trough, normal and peak phases of business cycles. Over the 1970-1998 period, these rates can be differentiated into three groupings; namely, years with negative and weak growth rates (growth rates less than 2.5%); years with "normal" growth of 2.5 % to 4.18%; and years with growth rates higher than 4.5%. Based on these cut-off values, there are 7 trough years, 14 normal years and 8 peak years in our 29-year sample.

The transition matrices corresponding to these three business cycle phases are reported in Table 5. Not surprisingly, the probabilities of default are highest and lowest during the trough and peak phases of the business cycle, respectively. Based on the t-test results reported in table 6, the probabilities are statistically different between the trough and peak phases of the business

cycle. Not only are all transition matrices different statistically but also these differences are most prominent for a comparison of the matrices for the normal and peak phases of the business cycle.

[Please place Tables 5 and 6 about here.]

These business-cycle-conditioned transition matrices are now used to determine n -year probabilities. The first step in doing so is to determine the relative frequency, π_{ij} , of going from state i (trough, normal or peak) to state j (trough, normal or peak) during the following year using historical data. For example, for the seven years when the initial phase was a trough, the following year was a peak year twice, a normal year three times and unchanged two times. Thus, $\pi_{\text{Trough,Trough}} = 2/7$, $\pi_{\text{Trough,Normal}} = 3/7$, and $\pi_{\text{Trough,Peak}} = 2/7$.¹¹ The next step is to calculate the unconditional default probabilities. The initial probability for any bond quote is drawn from that year's corresponding business cycle phase transition matrix. The relative frequencies and the transition matrices for the three business cycle phases are used to determine the expected unconditional default probabilities for the following years. To illustrate, take an Aa-rated bond quote for 1991 (a trough phase of the business cycle). If this bond has annual coupon payments and matures after 5 years, the one to five year default probabilities need to be determined. The one-year default is derived directly from the one-year trough transition matrix. The two-year unconditional default probabilities is determined by taking the first row from the default probabilities column in the two-year transition matrix starting from a trough phase:

$$M_{2 \text{ years}} = M_{\text{Trough}} \times [\pi_{\text{Trough,Trough}} M_{\text{Trough}} + \pi_{\text{Trough,Normal}} M_{\text{Normal}} + \pi_{\text{Trough,Peak}} M_{\text{Peak}}] \quad (3)$$

In (3), M is the ratings transition matrix, and π is the relative frequency as defined previously. A similar procedure is used to derive the three- to five-year unconditional default probabilities. For

¹¹ The remaining probabilities are: $\pi_{21} = 4/14$, $\pi_{22} = 8/14$, $\pi_{23} = 2/14$, $\pi_{31} = 1/8$, $\pi_{32} = 3/8$ and $\pi_{33} = 4/8$.

instance, the five-year default probability is derived from raising the appropriate value from the 2-year matrix obtained from equation (3) to the 4th power. These unconditional default probabilities then are used to derive the conditional ones when needed as is illustrated in footnote 5.¹²

5.2.3 The After-default Spot Curves¹³

In this section and unlike EGAM, we derive the after-default term structure of corporate spot rates from the actual prices of bonds using an approach similar to that used in section 4.1. Since we now account for the possibility that the bond could default before maturity, the spot rates obtained are lower than those reported in table 1 in section 4.1, where the difference is the default spread. The formula used to derive the after-default spot curves is:

$$\tilde{P}_t = \left\{ C \left(\sum_{m=1}^M d_{t,m} + d_{t,M} \right) \prod_{m=1}^M (1 - \lambda_m) + \sum_{m=1}^M \left\{ \left[C \sum_{i=1}^{m-1} d_{t,i} + \delta d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1 - \lambda_i) \right\} \right\} \quad (4)$$

In (4), \tilde{P}_t is the dirty price, C is the coupon, d is the discount factor, λ is the conditional probability of default, and δ is the recovery rate. Equation (1), which was used to derive the spot rates without accounting for default and tax effects, is easily obtained by assuming that $\lambda=0$ in (4). Based on the default spread findings reported in table 7,¹⁴ the default spreads over short-[long-]term periods are higher [lower] than those reported by EGAM. To illustrate, our 2-year

¹² Similar to the argument in footnote 6, the conditional default for any time interval Δt can be computed using Bayes' theorem as $[s(t) - s(t + \Delta t)] / \Delta t \times s(t)$ where $s(t)$ is the survival function.

¹³ Further support is found for our earlier finding that using a theoretical 10-year bond and the unaltered EGAM methodology to decompose the credit spread can result in erroneous spread (i.e., negative) measurements. These tests of robustness used the Nelson and Siegel (and Svensson) approaches and the unaltered EGAM methodology on our sample prior to the elimination of 10+-year bonds. Based on unreported results, some of the estimated default spreads for the Aa- and A-rated bonds were negative. The Nelson-Siegel-Svensson (NSS) yield curve is based on six parameters instead of four as in the Nelson-Siegel (NS) model, and it allows for two humps instead of one in estimating yield curves. The results are mixed in terms of the superiority of the NSS over the NS approach. Dionne et al (2004) also find negative default spread estimates for short maturities and highly related bonds when using a zero-coupon, ten-year theoretical bond to decompose the spread instead of a coupon-paying bond.

¹⁴ When incorporating the default spread to derive the after-default spot rates, the number of outliers and the root mean square errors do not change materially.

default spread for Aa-rated bonds are more than double those reported by EGAM, while our 10-year default spread is one-third of that reported by EGAM for the same period. Furthermore, our results support the literature findings that the size of the default spread is small. For example, for the 10-year period, our default spread estimates do not exceed 0.014%, 0.05% and 0.351% for the Aa-, A-, and Baa-rated bonds, respectively.

[Please place table 7 about here.]

5.3. Tax Spread Estimates

The expectation is that the after-tax yield on corporate bonds is higher than that of state-tax-free Treasury bonds all else held equal to compensate for the higher effective tax rate on the former in the US. To maintain comparability, an effective tax rate of 4% on Treasuries as in EGAM is used to calculate the magnitude of the tax spread.¹⁵ In the next section, the shortcomings of this measurement are addressed.

The following equation is used to compute the tax spread (for greater details, please see Appendix C):

$$e^{-(f_{t+1}^C - f_{t+1}^G)} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C} + \frac{C(1 - P_{t+1}) - (1 - a)P_{t+1}}{V_{t+1T} + C} (t_s)(1 - t_g) \quad (5)$$

Given the low effective tax rate, we expect and find in table 8 that the tax spread represents a small proportion of the credit spread (from around 0.4% to 8.4% for Aa- to Baa-rated bonds, respectively). These values are a little higher but consistent with EGAM.

[Please place table 8 about here.]

5.4. A Reexamination of the Tax Spread Estimates

The EGAM approach ignores many complexities of the tax system such as the different tax treatment of discount and premium bonds, and uses only a gross, exogenously determined

¹⁵ The effective tax rate is the state tax rate multiplied by one minus the federal tax rate, or $t_s(1 - t_g)$ in equation (3).

uniform tax rate of 4%. Instead, we incorporate more of the complexities of the actual tax system into our computations. Our approach is grounded mainly in the work of Green and Odegaard (1997) and Liu et al (2005). Appendix E provides the derivation of the after-tax term structure when the effect of personal and federal tax rates, the amortization of taxes, accrued interest taxes, issue dates, and the difference between premium and discount bond tax treatment are accounted for.¹⁶ By assuming that the taxes on income and capital gains are unknown in our optimization model, we can determine implied tax rates from the sample prices so that the tax rates are no longer constant by assumption as in EGAM. Our approach is consistent with the findings of Green (1993), Ang, Peterson and Peterson (1985), Skelton (1983), and Kryzanowski, Xu and Zhang (1995) that the implied marginal tax rates based on the spread between tax-free and taxable yields decrease with maturity.

Six parameters are estimated in our optimization model. These include the four parameters of the Nelson and Siegel approach, which are needed to derive the spot rates, the marginal income tax rate, and the capital gains tax rate. Based on table 7, our tax-spread estimates are generally lower than those reported by EGAM for short-term maturities and higher for the long-term maturities. Interestingly, our tax rate estimates materially exceed those reported by EGAM. We find that investors pay on average a tax of 5.43% on income and 5.44 % on capital gains for Aa rated bonds whereas they pay on average a tax of 6.34% on income and 6.43% on capital gains for A-rated bonds and pay about 8.94% on income and 9.33% on capital gains for Ba-rated bonds.

5.5 Risk Premium Estimates

¹⁶ After accounting for default and taxes, the number of outliers removed from the computation process becomes lower (804 bond prices) and the accuracy of our results is higher. The RMSEs for the Aa-, A-, and Baa-rated bonds become 1.1, 1.3, and 2.2, respectively.

Since systematic risk affects credit migration, which in turn could lead to a downgrading of the credit rating and higher uncertainty about recovery rates, systematic risk is expected to represent a significant portion of a credit spread. Many studies find a link between systematic risk and the credit spread. For example, Ericsson and Renault (2000) and Baraton and Cuillere (2001) show that the valuation of credit risk requires that one account for macroeconomic factors. Duffee (1998) finds that the correlation between credit spreads and the stock market is higher for high yield bonds than for low yield bonds. EGAM find that a large portion of the variation in credit spreads of corporate bonds is systematic in that the risk factors identified by Fama and French (1993) are priced.

Based on the size of the default and tax spreads, a considerable proportion of the credit spreads remains unexplained (specifically, the unexplained portion ranges from an average of 14.7 % for the Baa bonds to 45.45% for the Aa bonds). To test the relationship between the Fama and French (FF) systematic factors and the unexplained portion of the spread for the 120 term structures estimated earlier on a monthly basis for the ten-year period, the following relationship between spreads and the three FF factors is examined:

$$R_{t,t+1}^C - R_{t,t+1}^G = -m[(r_{t+1,m}^C - r_{t+1,m}^G) - (r_{t,m}^C - r_{t,m}^G)] = -m\Delta S_{t,m} \quad (6)$$

where $R_{t,t+1}^C$ and $R_{t,t+1}^G$ are the monthly returns on corporate and government constant maturity bonds maturing m periods later, respectively; m is the term-to-maturity of the bonds; r^C and r^G are the spot rates on corporate and government bonds, respectively; and $\Delta S_{t,m}$ is the monthly change in the credit spreads. Although Equation (6) relates spreads to returns, equation (6) needs to be extended to deal with what corresponds to only the unexplained portion of the total spread (i.e., after removing the portion explained by default and taxes). Doing such, equation (6) becomes:

$$R_{t,t+1}^{uc} - R_{t,t+1}^G = -m[(r_{t+1,m}^{uc} - r_{t+1,m}^G) - (r_{t,m}^{uc} - r_{t,m}^G)] = -m\Delta S_{t,m}^{uc} \quad (7)$$

where $\Delta S_{t,m}^{uc}$ and $R_{t,t+1}^{uc}$ are the unexplained portion of credit spread changes and returns, respectively; and all the other terms are as previously defined.¹⁷

Equation (7) is used to compute the unexplained excess returns based on the unexplained credit spreads.¹⁸ Similar to EGAM, we apply the Fama and French (1992) three-factor model to (7) to yield:

$$R_t^{uc} - R_t^G = \alpha + \beta_M R_{Mt} + \beta_{SMB} SMB_t + \beta_{HML} HML_t + e_t, t=1,2,..119 \quad (8)$$

where $R_t^{uc} - R_t^G$ is the excess unexplained monthly return, which is calculated from the monthly changes in the unexplained portion of spreads;¹⁹ R_{Mt} is the excess market return; SMB is the return on a portfolio of small stocks minus the return on a portfolio of large stocks; and HML is the return on a portfolio of stocks with high book to market values minus the return on a portfolio of stocks with low book to market values.

Based on the empirical results presented in table 9, we find that the explanatory power of this model across most maturities is insignificant, with the exception of the market factor across all maturities for the Baa bonds. These results support the findings of Collin-Dufresne et al (2001) who find that the FF factors are not significant and do not increase the overall explanatory power of their estimated model except for the 2 year maturity for the Aa and A ratings and for the 2 to 4 year maturities for the Baa rating. This contradicts the findings of EGAM who report an adjusted R^2 as high as 31% for the three-factor FF model. At least two possible explanations exist for the differences between our results and those of EGAM. The first is grounded in our rectification of

¹⁷ Equations (6) and (7) are derived more fully in Appendix D.

¹⁸ For example, if the change in the monthly unexplained spread is 0.1%, then the excess unexplained monthly return for the 2-year credit spread is $-2 \times 0.1\% = -0.2\%$. This is done for each month for the two to ten year unexplained credit spreads.

¹⁹ The unexplained portion of the spread is simply the credit spread minus the default spread minus the tax spread.

some of the shortcomings in the EGAM methodology used to determine the spreads. The second possible explanation is that the macroeconomic factors may be determinants of the unexplained spread in EGAM because they capture the conditional nature of default probabilities where the conditioning variable is the phase of the business cycle.

[Please place table 9 about here.]

6. THE ROLE OF ILLIQUIDITY

6.1 Measures of Illiquidity

Numerous measures of bond (il)liquidity are proposed in the literature. The liquidity measures range from direct measures based on quote and/or transaction data (such as the quoted or effective bid-ask spreads, quote or trade depth, quote or trade frequencies, trading volume and number of missing prices) to indirect measures based on bond-specific characteristics (such as issued amount, age, yield volatility, number of contributors, and yield dispersion). Since our data set does not contain bid and ask prices or volume traded, quote- and trade-based direct measures of liquidity are not used herein. Therefore, we use one direct and three indirect proxies to measure aggregate market liquidity.

Since all the liquidity proxies proposed in the literature are bond-specific, we used the average approach adopted for equities by Chordia et al. (2001) for equity markets to obtain our aggregate proxies.²⁰ We consider whether or not the aggregate liquidity indexes should include the eliminated bonds (callable, puttable, more than 10 year maturity, zero coupon and variable rate bonds), and whether the indexes should be differentiated by rating or industrial sector category. Intuitively, it seems most appropriate to form indexes based on the market from which each credit-spread curve is estimated (i.e., by rating and only including bonds in our final

²⁰Total value proxies are used for tests of robustness.

sample). However, since the factors that affect liquidity could be macro factors such as the business cycle, we would expect (and find that) the proxy that better captures such macro factors is the one with the largest bond market coverage.

Thus, we form the liquidity proxies based on three broad categorizations of the initial data in order to ensure that our liquidity findings are robust. The categories are: all traded bonds including treasuries and non-investment grade bonds (Cat1); the full corporate bond market (Cat2), or Cat1 minus treasuries; and industrial-sector bonds only (Cat3). For each of these three categories, three additional categories are formed but with the deletion of callable and puttable bonds and bonds with maturities exceeding ten years to use only the bonds in our final sample (Cat1a, Cat2a, Cat3a). And finally, six subsamples of bonds are formed for Aa-, A- and Baa-rated bonds from the two corporate bond categories (Cat2 and Cat2a) and the two industrial bond categories (Cat3 and Cat3a).²¹ This yields 18 measures of liquidity for each type of aggregate liquidity proxy.

The direct proxy of aggregate liquidity for a month is the relative frequency of monthly matrix prices to the total number of monthly corporate quotes during that month as captured in the Warga (1998) database. Any lack of liquidity in the corporate bond market should be reflected in the need for greater matrix pricing. Thus, this measure of thin trading should be inversely related with bond liquidity.

The first indirect proxy of aggregate market liquidity is the average dollar issued amount of bonds that were traded during the month. The dollar amount of the bond at the issue date is reported in the Warga and FISD databases. Since most investment banks rely on this measure to

²¹ For example, another three categories, Cat2aAA, Cat2aA and Cat2aBaa that represent the Aa-, A-, and Baa-rated corporate bonds, respectively, are formed from Cat2a.

form their bond indices,²² proponents of this measure argue that larger issues should trade more often than smaller issues since they are broadly disseminated among investors. Furthermore, Sarig and Warga (1989) and Amihud and Mendelson (1991) argue that bonds with smaller issued amounts tend to be absorbed in buy-and-hold portfolios more easily. Consequently, small issue bonds are not expected to generate much secondary market activity. Thus, both arguments lead to the expectation that a bond is more liquid with a larger issue size.

The second indirect proxy of aggregate market liquidity is the total age of bonds that traded during the month. This is obtained by finding the sum in years of the differences between the trading dates and the issue dates for all the bonds that traded during that month. The age of a bond is commonly used in the literature as an issuer-specific proxy of liquidity. The expected relationship between liquidity and age is that a bond becomes less liquid with increasing age because a higher portion of its outstanding position is held in the portfolios of buy-and-hold investors. Sarig and Warga (1989) observe that longer maturity bonds are more illiquid than shorter maturity bonds.

The third and final proxy of aggregate market liquidity is the mean of all the yield volatilities of bonds that traded during the month using yields starting 2 years earlier.²³ Since the inventory-cost component of bid-ask spreads is higher for greater yield volatility all else held constant, we expect illiquidity to increase with increasing yield volatility.²⁴

6.2 Explanatory Power of Illiquidity

²² For example, Lehman Brothers use this criterion for their Euro-Aggregate Corporate Bond index.

²³ To illustrate, take the month of January 1987. The mean of all the volatilities of each bond that traded during this month is calculated using historical yields from 1985.

²⁴ Hong and Warga (2000), Alexander et al. (2000), among others, use yield volatility as a proxy for uncertainty.

The following multiple regression is conducted first to determine if the portions of the credit spreads unexplained by default and taxes are related to the monthly aggregate (il)liquidity proxies for the 1987-1996 period:

$$S_{t,m}^{uc} = \beta_0 + \beta_1 Amount_{t,m} + \beta_2 Age_{t,m} + \beta_3 Matrix_{t,m} + \beta_4 Volatility_{t,m} + \varepsilon_{t,m}^{uc}, \quad (9)$$

where $S_{t,m}^{uc}$ is the portion of the credit spread unexplained by default and taxes for a term-to-maturity of m , which is measured either as the total credit spread minus the estimated default spread minus the estimated tax spread for a term-to-maturity of m , or alternatively, as the difference between the after-default and after-tax corporate and treasury term structures for a term-to-maturity of m ;

Amount is the average dollar issued amount of bonds in billions that traded during the month t ;

Age is the average age in thousands of years of bonds traded during the month t ;

Matrix is the relative frequency of monthly matrix prices to the total number of monthly corporate quotes during month t ;

Volatility is the average yield volatility in thousands of bonds quoted during month t ; and

$\varepsilon_{t,m}^{uc}$ is the error term with the usually assumed properties.

A comparison of the regression results for all aggregation categories shows that liquidity plays a major role in explaining the unexplained portion of credit spreads. A representative set of results is presented in table 10. These results are for regressions of the unexplained credit spreads for the Aa-, A- and Baa-rated bonds against the average liquidity proxies for Amount, Volatility and Age for their corresponding rating-specific aggregate corporate industrial bond indexes based on the initial sample of bonds (Cat3). Using the average value for Age allows for an

indirect capture of the size of the market as reflected by the number of bonds trading in each applicable rating category in each month.

[Please place table 10 about here.]

All reported regressions are strongly significant based on the F-test. All liquidity proxies are significant in most of the reported regressions and all categories. Therefore, aggregate liquidity is a major determinant of credit spreads, and plays sometimes even a more important role than default for the Aa rating category in the determination of the credit spread as is illustrated in table 11.

[Please place table 11 about here.]

7. CONCLUSION

In this paper, we reexamined the work of EGAM on the components of credit spreads for industrial investment grade bonds. We recomputed the default spread based on industrial-sector default probabilities conditional on the phases in the business cycle. We reexamined the tax spread by allowing for variable tax rates and by accounting for the intricacies of actual bond taxation and not just from the use of a gross estimate of the tax rate. Moreover, we derived the default and tax spreads by computing the after-default and after-tax spot rates instead of using a theoretical ten-year par value bond as in EGAM.

We obtained different estimates for the default and tax spreads where the latter proved to be more important in determining the credit spreads. However, unlike EGAM, we found that the three factors in the Fama and French model do not play any significant role in explaining the remaining (unexplained) portion of credit spreads. However, we found that a portion of the

unexplained spreads could be explained by market (il)liquidity using such proxies as issue amount, issue age and frequency of matrix pricing.

REFERENCES

- Amihud, Y., 2002. Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets* 5:1 (January), 31–56.
- Alexander, G. J., A. K. Edwards and M. G. Ferri, 2000. The determinants of trading volume of high-yield corporate bonds. *Journal of Financial Markets* 3, 177-204.
- Altman, E., and V. Kishore, 1998. Defaults and returns analysis through 1997. Working Paper, NYU Salomon Center.
- Amato, J., and E. Remolona, 2003. The credit spread puzzle. *BIS Quarterly Review* (December), 51-63.
- Amihud, Y., and H. Mendelson, 1991. Liquidity, maturity, and the yields on U.S. Treasury securities. *Journal of Finance* 46(4), 1411-1425.
- Ang, J., D. Peterson, and P. Peterson, 1985. Marginal tax rates: Evidence from nontaxable corporate bonds: A note. *Journal of Finance* 40 (1), 327-332.
- Baraton, X., and T. Cuillere, 2001. Modélisation des spreads de swap, credit special focus. Working Paper, Crédit Agricole Indosuez Credit and Bond Research.
- Carty, L., and J. Fons, 1994. Measuring changes in corporate credit quality. *The Journal of Fixed Income* 4, 27-4.
- Carty, L.V., 1997. Moody's rating migration and credit quality correlation, 1920-1996. Special comment, Moody's Investors Service, New York.
- Chordia, T., R. Roll, and V. R. Anshuman, 2001. Trading activity and expected stock returns. *Journal of Financial Economics* 59:1 (January), 3-32.
- Collin-Dufresne, P., R. Goldstein, and J.S. Martin, 2001. The determinants of credit spread changes. *Journal of Finance* 56, 2177-2208.
- Dionne, G., G. Gauthier, K. Hammami, M. Maurice, and J.G. Simonato, 2004. Default Risk on Corporate Yield Spreads. Working paper, HEC Montreal.
- Driessen, J., 2002. Is default event risk priced in corporate bonds? Working paper, University of Amsterdam.
- Duffee, G., 1998. The relationship between treasury yields and corporate bond yield spreads. *Journal of Finance* 53, 2225-2241.
- Elton, E., M. Gruber, D. Agrawal, and C. Mann, 2001. Explaining the rate spread on corporate bonds. *Journal of Finance* 56, 247-277.
- Eom, Y., J. Helwege, and J. Huang. Structural models of corporate bond pricing: An empirical analysis. Forthcoming: *Review of Financial Studies*, 2004, 17, 499-544.
- Ericsson, J., and O. Renault, 2001. Credit and liquidity risk. Working paper, McGill University.
- Fama, E., and K. French, 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427-465.

- Green, R., 1983. A simple model of the taxable and tax-exempt yield curves. *Review of Financial Studies* 6 (2), 233-264.
- Green, R. C., and B. A. Odegaard, 1997. Are there tax effects in the U.S. Government bonds? *Journal of Finance* 52, 609-633.
- Hong, G., and A. Warga, 2000. An empirical study of bond market transactions. *Financial Analysts Journal* 56(2), 32-46.
- Houweling, P., A. Mentink, and T. Worst, 2003. Comparing possible proxies for corporate bond liquidity. SSRN working paper.
- Huang, J.-Z., and M. Huang, 2003. How much of the corporate-treasury yield spread is due to credit risk? Working paper, Penn State and Stanford Universities.
- Kryzanowski, L., and K. Xu, 1997. Long-term equilibria of taxable and tax-exempt bonds. *International Review of Economics and Finance* 6:2, 119-143.
- Kryzanowski, L., K. Xu and H. Zhang, 1995. Determinants of the decreasing term structure of relative yield spreads for taxable and tax-exempt bonds. *Applied Economics* 27, 583-590.
- Leland, H., and K. Toft, 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance* 51, 987-1019.
- Liu, S., J. Shi, J. Wang, and C. Wu, 2005. How much of the corporate bond spread is due to personal taxes? Working paper, Syracuse, Youngstown State and Marshall Universities.
- Merton, R. C., 1974. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29, 449-470.
- Morris, C., R. Neal, and D. Rolph, 2000. Interest rates and credit spread dynamics. Working Paper, Indiana University.
- Nelson, C., and A. Siegel, 1987. Parsimonious modeling of yield curves. *Journal of Business* 60, 473-89.
- Nickell, P., W. Perraudin, and S. Varotto, 2000. Stability of ratings transitions. *Journal of Banking and Finance* 24, 203-227.
- Sarig, O., and A. Warga, 1989. Bond price data and bond market liquidity. *Journal of Financial and Quantitative Analysis* 24(3), 367-378.
- Schultz, P., 2001. Corporate bond trading costs and practices: A peek behind the curtain. *Journal of Finance* 56(2), 677-698.
- Skelton, J., 1983. Banks, firms and the relative pricing of tax-exempt and taxable bonds. *Journal of Financial Economics* 12, 343-355.
- Svensson, L., 1994. Estimating and interpreting forward interest rates: Sweden 1992-4. Working paper, Centre for Economic Policy Research.

Table 1. Measured Spreads from Treasuries and Average Root Mean Squared Errors

Panel A reports the mean of monthly credit spreads from Treasuries for Aa-, A-, and Baa-rated bonds in the industrial sector for the 1987-1996 period. The treasury and spot rates are derived using the Nelson and Siegel approach and do not account for default and taxes. Treasury average spot rates are reported as annualized spot rates (in %). Corporate credit spreads are reported as the difference between the derived corporate spot rates and the derived treasury spot rates. The corporate term structures are those with the lower error and the least number of outliers. Panel B reports the average root mean squared errors of the differences at a monthly frequency between the theoretical prices derived from using the theoretical spot rates and the actual bond prices for Treasuries and Aa-, A- and Baa-rated coupon-paying corporate bonds for the entire 1987-1996 period and for first and last 5-year periods. Root mean squared errors are measured in cents per \$100.

Maturities/ Period	Our Results				EGAM Results			
	Treasuries	Aa	A	Baa	Treasuries	Aa	A	Baa
Panel A: Measured spreads from treasuries (in %)								
2	6.128	0.484	0.522	0.975	6.414	0.414	0.621	1.167
3	6.305	0.510	0.566	1.052	6.689	0.419	0.680	1.205
4	6.460	0.530	0.603	1.107	6.925	0.455	0.715	1.210
5	6.598	0.546	0.633	1.147	7.108	0.493	0.738	1.205
6	6.721	0.559	0.659	1.177	7.246	0.526	0.753	1.199
7	6.831	0.568	0.680	1.198	7.351	0.552	0.764	1.193
8	6.930	0.575	0.698	1.212	7.432	0.573	0.773	1.188
9	7.019	0.579	0.712	1.221	7.496	0.589	0.779	1.184
10	7.099	0.582	0.724	1.224	7.548	0.603	0.785	1.180
Panel B: Average root mean squared errors (cents per \$100)								
1987-1996	0.362	0.933	1.027	1.570	0.210	0.728	0.874	1.516
1987-1991	0.602	1.303	1.434	1.880	0.185	0.728	0.948	1.480
1991-1996	0.232	0.682	0.781	1.517	0.234	0.727	0.800	1.552

Table 2. Average One-Year Rating All-sectors and Industrial Transition Matrices and All-sectors Recovery Rates

This table presents the average rating transition probabilities (in %) for a one-year tracking horizon. The all-sectors probabilities as reported in panel A are taken from Carty and Fons (1994). The industrial-sector probabilities are based on the ratings of industrial, US domicile, senior unsecured corporate debt as found in the Moody's DRS database for the 1970-1998 period. These industrial sector probabilities do not account for the effect of the business cycle. Each entry in a row shows the probability that a bond with a rating shown in the first column ends up one year later in the category shown in the column headings. Panel C reports the recovery rates in (%) for each ratings category from Altman and Kishore (1998).

Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Default
Panel A: All-sectors transition matrix								
Aaa	91.90	7.39	0.72	0.00	0.00	0.00	0.00	0.00
Aa	1.13	91.26	7.09	0.31	0.21	0.00	0.00	0.00
A	0.10	2.56	91.19	5.33	0.62	0.21	0.00	0.00
Baa	0.00	0.21	5.36	87.94	5.46	0.83	0.10	0.10
Ba	0.00	0.11	0.43	5.00	85.12	7.33	0.43	1.59
B	0.00	0.11	0.11	0.54	5.97	82.19	2.17	8.90
Caa	0.00	0.44	0.44	0.87	2.51	5.90	67.80	22.05
Panel B: Industrial-sector transition matrix with business cycle effects not accounted for								
Aaa	93.29	6.25	0.46	0.00	0.00	0.00	0.00	0.00
Aa	1.12	90.34	8.09	0.23	0.18	0.03	0.00	0.00
A	0.07	1.75	92.80	4.64	0.55	0.18	0.02	0.00
Baa	0.03	0.10	4.55	89.02	4.87	0.90	0.32	0.21
Ba	0.00	0.02	0.65	6.68	85.32	6.14	0.83	0.36
B	0.00	0.00	0.04	0.31	6.67	84.31	5.82	2.85
Caa	0.00	0.00	0.00	0.66	1.59	4.58	80.94	12.23
Panel C: All-sectors recovery rates								
Recovery	68.34	59.59	60.63	49.42	39.05	37.54	38.02	0.00

Table 3. Evolution of Default Probabilities

This table reports the conditional default probabilities in (%) that a bond with either an Aa, A or Baa rating defaults after n number of years. These probabilities are derived using the one-year all-sectors transition matrix reported by Carty and Fons (1994), and the industrial transaction matrix derived from this all-sectors transition matrix.

Year	All-sectors default probabilities			Industrial default probabilities		
	Aa	A	Baa	Aa	A	Baa
1	0.000	0.000	0.103	0.000	0.000	0.208
2	0.004	0.034	0.274	0.002	0.019	0.268
3	0.011	0.074	0.441	0.006	0.041	0.330
4	0.022	0.121	0.598	0.012	0.065	0.392
5	0.036	0.172	0.743	0.020	0.091	0.454
6	0.053	0.225	0.874	0.030	0.118	0.514
7	0.073	0.280	0.991	0.041	0.147	0.571
8	0.095	0.336	1.095	0.054	0.177	0.625
9	0.120	0.391	1.186	0.068	0.207	0.676
10	0.146	0.446	1.264	0.083	0.238	0.723

Table 4. Average Default Spreads Assuming Risk Neutrality

This table reports the average default spreads of corporate spot rates over government spot rates (in %) when taxes are assumed to be zero. These default spreads are computed under the assumption of risk neutrality using equation (2) and after accounting for the recovery and default rates reported in tables 2 and 3, respectively. The default rates are derived from both an all-sectors transition matrix (TM) and an industrial-sector TM.

Years	Default spreads, all-sectors TM						Default spreads, industrial-sector TM		
	Our Results			EGAM Results			Our Results		
	Aa	A	Baa	Aa	A	Baa	Aa	A	Baa
2	0.001	0.007	0.103	0.004	0.053	0.145	0.000	0.004	0.130
3	0.002	0.016	0.148	0.008	0.063	0.181	0.001	0.009	0.146
4	0.004	0.025	0.191	0.012	0.074	0.217	0.002	0.013	0.162
5	0.006	0.035	0.232	0.017	0.084	0.252	0.004	0.019	0.178
6	0.009	0.045	0.272	0.023	0.095	0.286	0.005	0.024	0.194
7	0.012	0.056	0.309	0.028	0.106	0.319	0.007	0.030	0.210
8	0.016	0.067	0.344	0.034	0.117	0.351	0.009	0.035	0.226
9	0.020	0.078	0.377	0.041	0.128	0.380	0.011	0.041	0.242
10	0.025	0.090	0.408	0.048	0.140	0.409	0.014	0.048	0.257

Table 5. Average One-Year Rating Transition Matrices for the Industrial Sector During Normal, Trough, and Peak Phases of the Business Cycle

This table presents the average rating transition probabilities in (%) for a one-year tracking horizon as estimated from Moody's DRS database for the 1970-1997 period. These estimates are based on the ratings of industrial, US domicile, senior unsecured corporate debt during normal, trough and peak phases of the business cycle. The state of the business cycle is identified using the growth in GNP rates as a benchmark. Each entry in a row shows the probability that a bond with a rating shown in the first column ends up one year later in the category shown in the subsequent column headings.

Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Default
Panel A: Normal Phases of the Business Cycle								
Aaa	93.55	6.05	0.40	0.00	0.00	0.00	0.00	0.00
Aa	1.01	87.04	11.10	0.41	0.37	0.07	0.00	0.00
A	0.07	1.43	91.79	5.27	1.08	0.36	0.00	0.00
Baa	0.07	0.16	4.55	88.00	5.13	1.44	0.36	0.30
Ba	0.00	0.05	0.33	5.86	85.26	7.17	1.03	0.30
B	0.00	0.00	0.09	0.40	6.30	83.70	6.57	2.93
Caa	0.00	0.00	0.00	1.37	2.92	8.05	78.46	9.19
Panel B: Trough Phases of the Business Cycle								
Aaa	94.32	5.28	0.40	0.00	0.00	0.00	0.00	0.00
Aa	1.47	90.89	7.64	0.00	0.00	0.00	0.00	0.00
A	0.07	2.32	92.35	5.10	0.07	0.00	0.07	0.00
Baa	0.00	0.10	5.55	86.65	6.92	0.47	0.10	0.20
Ba	0.00	0.00	0.62	5.48	85.71	6.50	0.81	0.88
B	0.00	0.00	0.00	0.33	5.95	81.47	6.57	5.68
Caa	0.00	0.00	0.00	0.00	0.00	1.31	81.24	17.45
Panel C: Peak Phases of the Business Cycles								
Aaa	91.93	7.45	0.63	0.00	0.00	0.00	0.00	0.00
Aa	1.02	95.62	3.23	0.13	0.00	0.00	0.00	0.00
A	0.06	1.80	94.96	3.14	0.05	0.00	0.00	0.00
Baa	0.00	0.00	3.68	92.87	2.64	0.31	0.44	0.06
Ba	0.00	0.00	1.25	9.16	85.08	4.03	0.48	0.00
B	0.00	0.00	0.00	0.15	7.93	87.85	3.83	0.24
Caa	0.00	0.00	0.00	0.00	0.63	1.39	84.73	13.26

Table 6. T-tests for the Differences Between the Means of the One-year Rating Transition Matrices for the Industrial Sector During Normal, Trough and Peak Phases of the Business Cycle

This table presents the results of various t-tests for the differences between the means of the different transition matrices reported in table 5. Differences that are significantly different from zero at the 10, 5 and 1% levels are indicated by *, ** and ***, respectively.

Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Default
Panel A: T-statistics for the differences in means for the peak and normal transition matrices								
Aaa	-0.46	0.44	0.32	-	-	-	-	-
Aa	0.01	3.15***	-3.00***	-0.82	-1.38	-0.75	-	-
A	-0.09	0.59	1.94*	-1.87*	-2.14**	-1.57	-	-
Baa	-0.75	-0.75	-0.80	2.08**	-1.81	-2.03	0.21	-1.66
Ba	-	-0.75	1.96*	1.89*	-0.09	-1.65	-0.87	-1.46
B	-	-	-0.99	-0.78	0.52	1.19	-1.15	-2.28**
Caa	-	-	-	-0.75	-0.93	-2.03*	1.15	0.86
Panel B: T-statistics for the differences in means for the peak and trough transition matrices								
Aaa	-0.60	0.63	0.30	-	-	-	-	-
Aa	-0.51	1.96*	-1.66	0.93	-	-	-	-
A	-0.14	-0.66	1.01	-0.87	-0.33	-	-1.08	-
Baa	-	-1.08	-1.26	1.88*	-1.73	-0.46	0.90	-1.04
Ba	-	-	0.72	1.46	-0.17	-0.66	-0.50	-1.63
B	-	-	-	-0.71	0.42	0.99	-0.67	-2.14**
Caa	-	-	-	-	0.93	0.19	1.09	-0.20
Panel C: T-statistics for the differences in means for the normal and trough transition matrices								
Aaa	-0.27	0.29	0.00	-	-	-	-	-
Aa	-0.59	-1.28	1.16	1.18	1.29	0.70	-	-
A	-0.08	-1.24	-0.25	0.09	1.94*	1.47	-1.45	-
Baa	0.70	0.23	-0.74	0.46	-0.86	1.62	0.86	0.58
Ba	-	0.70	-0.64	0.21	-0.14	0.21	0.34	-1.26
B	-	-	0.92	0.20	0.14	0.57	0.00	-1.22
Caa	-	-	-	0.70	1.16	1.97*	0.24	-1.24

Table 7. Average Default and Tax Spreads Derived Using After-default and After-tax Spot Rates

Panel A reports the average default spreads (in %) calculated as the differences between the pre-default and tax corporate spot rates and their after-default but pre-tax counterparts. Panel B reports the average tax spreads (in %) calculated as the difference between the after-default spot rates, which are reported in panel A, and the after-default and tax spot rates, which are reported in this panel. The after-default and tax spot rates are derived after accounting for default and tax price effects.

Years	Aa	A	Baa	Aa	A	Baa
Panel A: After-default spot rates and average default spreads						
	After-default Spot Rates (in %)			Average Default Spreads (in %)		
2	6.604	6.637	6.609	0.009	0.013	0.228
3	6.808	6.854	6.875	0.007	0.017	0.264
4	6.984	7.041	7.093	0.006	0.021	0.289
5	7.138	7.205	7.278	0.006	0.026	0.308
6	7.272	7.349	7.437	0.007	0.030	0.322
7	7.391	7.476	7.575	0.008	0.035	0.333
8	7.495	7.587	7.695	0.010	0.040	0.341
9	7.586	7.686	7.800	0.012	0.045	0.347
10	7.667	7.773	7.892	0.014	0.050	0.351
Panel B: After-tax spot rates and average tax spreads						
	After-tax Spot Rates (in %)			Average Tax Spreads (in %)		
2	6.101	6.102	6.181	0.255	0.278	0.428
3	6.304	6.306	6.383	0.299	0.331	0.491
4	6.479	6.481	6.555	0.329	0.372	0.538
5	6.633	6.635	6.703	0.351	0.406	0.575
6	6.769	6.772	6.831	0.369	0.433	0.606
7	6.889	6.894	6.942	0.383	0.455	0.633
8	6.996	7.003	7.040	0.394	0.473	0.656
9	7.092	7.100	7.125	0.403	0.487	0.675
10	7.176	7.187	7.200	0.410	0.499	0.693

Table 8. Average Tax Spreads Assuming Risk Neutrality

This table reports the average tax spreads of corporate spot rates over government spot rates (in %) when the effective tax rate is assumed to be equal to 4% as in Elton et al. (2001). These tax spreads are computed under the assumption of risk neutrality using equation (3). The EGAM results also are presented to facilitate comparison.

Years	Our Results			EGAM Results		
	Aa	A	Baa	Aa	A	Baa
2	0.353	0.358	0.509	0.296	0.344	0.436
3	0.358	0.368	0.531	0.301	0.354	0.47
4	0.363	0.378	0.552	0.305	0.364	0.504
5	0.368	0.388	0.574	0.309	0.374	0.537
6	0.374	0.398	0.595	0.314	0.383	0.569
7	0.381	0.409	0.615	0.319	0.393	0.600
8	0.387	0.419	0.634	0.324	0.403	0.629
9	0.394	0.430	0.653	0.329	0.413	0.657
10	0.402	0.440	0.670	0.335	0.423	0.683

Table 9. Relationship Between Returns and Fama-French Risk Factors

This table reports the results of the regressions of returns due to changes in the unexplained spreads on industrial corporate bonds and returns on the Fama-French (FF) risk factors (the market excess return, the small minus big factor, and the high minus low book-to-market factors). The FF-factors are obtained from French's online data library. The p-values for the parameter estimates are reported in the parentheses next to their coefficient estimates. The last column reports the p-values for the regressions. ^a, ^b and ^c indicate statistical significance at the 0.10, 0.05 and 0.01 levels, respectively.

Maturities	Constant	Fama-French Risk Factors			Adj-R ² (%)	P-value
		Market	SMB	HTML		
Panel A: Industrial Aa-rated bonds						
2	-0.005(0.93)	0.005(0.72)	-0.059(0.01) ^c	-0.025(0.33)	2.67	0.070 ^a
3	-0.004(0.96)	0.002(0.94)	-0.075(0.02) ^b	-0.026(0.48)	1.36	0.137
4	-0.004(0.97)	-0.001(0.96)	-0.087(0.04) ^b	-0.023(0.62)	0.52	0.208
5	-0.004(0.97)	-0.004(0.91)	-0.096(0.06) ^a	-0.018(0.76)	-0.11	0.281
6	-0.005(0.97)	-0.006(0.88)	-0.105(0.09) ^a	-0.012(0.87)	-0.59	0.352
7	-0.007(0.97)	-0.008(0.86)	-0.113(0.12)	-0.004(0.96)	-0.95	0.415
8	-0.009(0.97)	-0.010(0.86)	-0.121(0.15)	0.004(0.97)	-1.22	0.467
9	-0.012(0.96)	-0.010(0.87)	-0.129(0.17)	0.011(0.92)	-1.43	0.511
10	-0.016(0.95)	-0.009(0.90)	-0.139(0.20)	0.019(0.88)	-1.59	0.547
Panel B: Industrial A-rated bonds						
2	-0.023(0.64)	0.024(0.08) ^a	-0.051(0.01) ^c	0.009(0.70)	3.47	0.046 ^b
3	-0.026(0.72)	0.025(0.20)	-0.053(0.08) ^a	0.016(0.64)	0.35	0.225
4	-0.025(0.80)	0.023(0.39)	-0.048(0.24)	0.020(0.66)	-1.64	0.558
5	-0.021(0.87)	0.017(0.63)	-0.039(0.45)	0.023(0.70)	-2.69	0.830
6	-0.015(0.93)	0.007(0.87)	-0.030(0.65)	0.025(0.74)	-3.13	0.942
7	-0.006(0.97)	-0.005(0.92)	-0.021(0.80)	0.026(0.78)	-3.23	0.965
8	0.003(0.99)	-0.019(0.76)	-0.012(0.90)	0.027(0.81)	-3.19	0.955
9	0.011(0.97)	-0.033(0.65)	-0.006(0.96)	0.028(0.82)	-3.08	0.932
10	0.019(0.95)	-0.046(0.58)	-0.001(0.99)	0.030(0.83)	-2.97	0.905
Panel C: Industrial Baa-rated bonds						
2	-0.019(0.75)	0.037(0.03) ^b	-0.070(0.01) ^c	-0.024(0.40)	6.80	0.007 ^c
3	-0.035(0.70)	0.057(0.02) ^b	-0.086(0.02) ^b	-0.016(0.71)	4.80	0.022 ^b
4	-0.052(0.69)	0.076(0.03) ^b	-0.098(0.06) ^a	-0.004(0.95)	2.91	0.061 ^a
5	-0.071(0.68)	0.096(0.04) ^b	-0.109(0.12)	0.009(0.91)	1.65	0.118
6	-0.092(0.67)	0.116(0.05) ^a	-0.120(0.18)	0.023(0.82)	0.86	0.175
7	-0.114(0.67)	0.137(0.06) ^a	-0.130(0.24)	0.038(0.76)	0.37	0.223
8	-0.138(0.66)	0.160(0.07) ^a	-0.140(0.29)	0.054(0.72)	0.08	0.257
9	-0.164(0.66)	0.184(0.07) ^a	-0.148(0.33)	0.070(0.68)	-0.08	0.278
10	-0.191(0.65)	0.210(0.07) ^a	-0.156(0.36)	0.088(0.66)	-0.14	0.286

Table 10. Relationship Between Unexplained Credit Spreads and Aggregate Liquidity Proxies for the 1987-1996 Period

This table reports the results of the regressions of unexplained credit spreads (in %) (i.e., the portion not explained by default and taxes) for years two through ten. The variable “Amount” represents the average dollar amount (in billions) of issues for the bonds traded during the month. The variable “Age” represents the average age of bonds (in thousands of years) traded during the month. The variable “Matrix” is the relative frequency of matrix prices during the month. The variable “Volatility” is the average yield volatility for all bonds (in thousands) quoted during the month. The p-values for the parameter estimates are reported in the parentheses next to their coefficient estimates. The last column reports the p-values for the regressions. ^a, ^b and ^c indicate statistical significance at the 0.10, 0.05 and 0.01 levels, respectively.

Maturity	Coefficient Estimates (p-values in parentheses)					Adj. R ² (%)	P-value
	Constant	Amount	Age	Matrix	Volatility		
Panel A: Industrial Aa-rated bonds							
2	-0.25(0.412)	0.28 (0.556)	10.73(0.333)	-0.18(0.469)	3.20(0.000) ^c	11.99	0.056
3	-0.45(0.119)	0.57 (0.208)	12.10(0.251)	0.01(0.956)	3.52(0.000) ^c	17.03	0.002
4	-0.58(0.038) ^b	0.76 (0.084) ^a	12.21(0.233)	0.15(0.527)	3.79(0.000) ^c	22.03	0.000
5	-0.69(0.015) ^b	0.90 (0.040) ^b	12.42(0.225)	0.25(0.279)	3.97(0.000) ^c	25.57	0.000
6	-0.77(0.007) ^c	1.02 (0.022) ^b	13.05(0.207)	0.34(0.148)	4.08(0.000) ^c	27.58	0.000
7	-0.84(0.004) ^c	1.12 (0.014) ^b	14.14(0.180)	0.42(0.080) ^a	4.10(0.000) ^c	28.46	0.000
8	-0.90(0.002) ^c	1.20 (0.009) ^c	15.61(0.147)	0.50(0.044) ^b	4.05(0.000) ^c	28.57	0.000
9	-0.96(0.002) ^c	1.28 (0.007) ^c	17.38(0.112)	0.56(0.025) ^b	3.96(0.000) ^c	28.15	0.000
10	-1.01(0.001) ^c	1.34 (0.005) ^c	19.39(0.082) ^a	0.63(0.015) ^b	3.81(0.000) ^c	27.33	0.000
Panel B: Industrial A-rated bonds							
2	0.70(0.013) ^b	-0.92 (0.036) ^b	-6.83(0.503)	-0.59(0.012) ^b	0.21(0.796)	5.19	0.025
3	0.85(0.001) ^c	1.06 (0.009) ^c	-12.95(0.167)	-0.62(0.004) ^c	-0.27(0.715)	6.48	0.013
4	0.94(0.000) ^c	-1.14 (0.004) ^c	-17.16(0.063) ^a	-0.64(0.003) ^c	-0.61(0.406)	7.78	0.006
5	1.01(0.000) ^c	-1.18 (0.004) ^c	-20.05(0.034) ^b	-0.65(0.003) ^c	-0.86(0.251)	8.55	0.004
6	1.05(0.000) ^c	-1.20 (0.005) ^c	-21.96(0.027) ^b	-0.65(0.004) ^c	-1.06(0.177)	8.74	0.004
7	1.07(0.000) ^c	-1.21 (0.007) ^c	-23.14(0.026) ^b	-0.64(0.007) ^c	-1.22(0.138)	8.53	0.004
8	1.08(0.000) ^c	-1.20 (0.010) ^c	-23.77(0.029) ^b	-0.63(0.012) ^b	-1.35(0.118)	8.06	0.005
9	1.07(0.001) ^c	-1.18 (0.014) ^b	-23.97(0.033) ^b	-0.60(0.019) ^b	-1.45(0.106)	7.42	0.008
10	1.06(0.001) ^c	-1.15 (0.020) ^b	-23.85(0.040) ^b	-0.57(0.031) ^b	-1.52(0.099) ^a	6.68	0.011
Panel C: Industrial Baa-rated bonds							
2	0.44(0.196)	-0.82 (0.127)	-2.13(0.866)	-0.53(0.068) ^a	2.70(0.008) ^c	14.47	0.000
3	0.15(0.647)	-0.39 (0.448)	2.87(0.811)	-0.21(0.444)	2.77(0.004) ^c	15.08	0.000
4	-0.10(0.774)	-0.03 (0.955)	7.46(0.543)	0.06(0.842)	2.76(0.005) ^c	14.90	0.000
5	-0.31(0.376)	0.28 (0.608)	12.30(0.340)	0.29(0.330)	2.67(0.010) ^c	14.44	0.000
6	-0.50(0.174)	0.00(0.334)	17.39(0.200)	0.49(0.112)	2.50(0.022) ^b	13.95	0.000
7	-0.68(0.081) ^a	0.81 (0.183)	22.61(0.113)	0.68(0.037) ^b	2.28(0.045) ^b	13.58	0.000
8	-0.84(0.039) ^b	1.04 (0.102)	27.86(0.061) ^a	0.86(0.012) ^b	2.02(0.088) ^a	13.39	0.000
9	-0.98(0.020) ^b	1.24 (0.058) ^a	33.07(0.032) ^b	1.02(0.004) ^c	1.72(0.158)	13.38	0.000
10	-1.11(0.010) ^c	1.43 (0.034) ^a	38.18(0.016) ^b	1.16(0.001) ^c	1.41(0.261)	13.54	0.000

Table 11. Summary of the Determinants of Credit Spreads

This table summarizes the findings of our paper. We report the percentage explanatory power of each factor that we have investigated in this paper. The default and tax spreads explanatory power was computed directly from dividing the default and tax spreads by the credit spreads. On the other hand, the explanatory powers of the Fama and French risk premiums and liquidity premiums were computed by multiplying the adjusted R^2 of the regressions by the unexplained (after tax and default) portion of the credit spreads.

Maturity/Rating	Default Spreads			Tax Spreads			FF Risk Premiums			Liquidity Premiums		
	Aa	A	Baa	Aa	A	Baa	Aa	A	Baa	Aa	A	Baa
2	1.86	2.49	23.38	52.69	53.26	43.90	1.21	1.54	2.22	5.45	2.30	4.73
3	1.37	3.00	25.10	58.63	58.48	46.67	0.00	0.00	1.36	6.81	2.50	4.26
4	1.13	3.48	26.11	62.08	61.69	48.60	0.00	0.00	0.74	8.11	2.71	3.77
5	1.10	4.11	26.85	64.29	64.14	50.13	0.00	0.00	0.00	8.85	2.71	3.32
6	1.25	4.55	27.36	66.01	65.71	51.49	0.00	0.00	0.00	9.03	2.60	2.95
7	1.41	5.15	27.80	67.43	66.91	52.84	0.00	0.00	0.00	8.87	2.38	2.63
8	1.74	5.73	28.14	68.52	67.77	54.13	0.00	0.00	0.00	8.50	2.14	2.38
9	2.07	6.32	28.42	69.60	68.40	55.28	0.00	0.00	0.00	7.97	1.88	2.18
10	2.41	6.91	28.68	70.45	68.92	56.62	0.00	0.00	0.00	7.42	1.61	1.99

Appendix A The Nelson and Siegel Approach

The original motivation for this modeling method was a desire to create a model that could capture the range of shapes generally seen in yield curves; namely, monotonic and s-shapes. Nelson and Siegel assume that the instantaneous forward rate at any time t could be captured by a sequence of exponential terms that are represented by the following functional form:²⁵

$$f(t) = \beta_0 + \beta_1 \exp(-t/\tau_1) + \beta_2 (t/\tau_1) \exp(-t/\tau_1) \quad (\text{A1})$$

Since spot rates can be represented as the average of the relevant instantaneous forward rates, Nelson and Siegel derive the spot rate function as:

$$r(t) = \beta_0 + \beta_1 \left[\frac{1 - \exp(-t/\tau_1)}{t/\tau_1} \right] + \beta_2 \left[\frac{1 - \exp(-t/\tau_1)}{t/\tau_1} - \exp(-t/\tau_1) \right] \quad (\text{A2})$$

This model has four parameters that must be estimated, β_0 , β_1 , β_2 and τ_1 . The expected impact of these parameters on the shape of the spot rate function curve is as follows: β_0 depicts the long-term component because it is the limiting value of $r(t)$ as maturity gets larger. The implied short-term rate of interest is $\beta_0 + \beta_1$ because this is the limiting value, as maturity tends to zero. β_1 defines the basic speed with which the curve tends towards its long-term trend, and a positive [negative] sign for β_1 indicates a negative [positive] slope for the curve. τ_1 specifies the position of the hump or U-shape in the curve.

Appendix B

Measuring the Default Premium in a Risk-Neutral World Without State Taxes

In a risk neutral world, the value of a corporate bond is the certainty equivalent cash flows discounted back to time zero at the government bond rate. Consequently, the value of a two-year corporate bond could be expressed as:

$$V_{12} = [C(1-P_2) + aP_2 + 1(1-P_2)] e^{-f_{12}^G} \quad (\text{B1})$$

²⁵The instantaneous forward rate can be defined as the marginal cost of borrowing for an infinitely short period of time.

where C is the coupon rate; P_t is the probability of bankruptcy in period t conditional on surviving an earlier period; a is the recovery rate assumed to be a constant percentage of the principal in each period; f_{t+1}^G is the risk-free forward rate as of time 0 from t to $t+1$; and V_{tT} is the value of a T period bond at time conditional on surviving an earlier period.

Similarly, the time zero value could be expressed as:

$$V_{02} = [C(1-P_2) + aP_2 + 1(1-P_2)] e^{-f_{01}^G} \quad (B2)$$

On the other hand, the same bond could be expressed in terms of promised cash flows and corporate forward rates at year 1 by:

$$V_{12} = (C+1)e^{-f_{12}^C} \quad (B3)$$

where f_{t+1}^C is the forward rate from t to $t+1$ for corporate bonds. Using the same logic, the time zero value could be expressed as:

$$V_{02} = (C+1)e^{-f_{12}^C} \quad (B4)$$

Equating the two values of V_{12} and rearranging yields a forward spread of:

$$e^{-(f_{12}^C - f_{12}^G)} = (1 - P_2) + [aP_2 / (1 + C)] \quad (B5)$$

Equating these expressions for V_{02} yields a forward rate spread of:

$$e^{-(f_{01}^C - f_{01}^G)} = (1 - P_2) + [aP_2 / (V_{12} + C)] \quad (B6)$$

Generalizing (B5) and (B6), the difference in forward rates at period t is:

$$e^{-(f_{t+1}^C - f_{t+1}^G)} = (1 - P_{t+1}) + [aP_{t+1} / (V_{t+1T} + C)] \quad (B7)$$

where $V_{TT} = 1$.

Appendix C

Measuring State Taxes

The same argument as in the previous appendix is used to reflect the tax spread effect but now the tax rates are included in the equations. Consequently, the V_{01} is expressed in terms of risk neutrality as:

$$V_{01} = [C(1-P_1)(1-t_s(1-t_g)) + aP_1 + (1-a)P_1 t_s(1-t_g) + (1-P_1)] e^{-f_{01}^G} \quad (C1)$$

where the additional terms t_s and t_g are the state and federal tax rates, respectively, and all the other terms are defined as in appendix B.

Also, V_{01} can be expressed in terms of promised cash flows as:

$$V_{01} = (C+1) e^{-f_{01}^C} \quad (C2)$$

Equating the two expressions yields:

$$e^{-(f_{01}^C - f_{01}^G)} = (1-P_1) + \frac{aP_1}{1+C} + \frac{C(1-P_1) - (1-a)P_1}{1+C} (t_s)(1-t_g) \quad (C3)$$

In general, the forward rate spread becomes:

$$e^{-(f_{t+1}^C - f_{t+1}^G)} = (1-P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C} + \frac{C(1-P_{t+1}) - (1-a)P_{t+1}}{V_{t+1T} + C} (t_s)(1-t_g) \quad (C4)$$

Appendix D

The Relationship Between Returns and Spreads

Let $r_{t,m}^C$ and $r_{t,m}^G$ be the spot rates on a corporate bond and a government bond, respectively, that mature at period m . Then the price of a corporate and a government zero-coupon bond with face value equal to one dollar respectively is:

$$P_{t,m}^C = e^{-r_{t,m}^C \cdot m} \quad \text{and} \quad P_{t,m}^G = e^{-r_{t,m}^G \cdot m} \quad (D1)$$

One month later the prices for the corporate and government bond respectively become:

$$P_{t+1,m}^C = e^{-r_{t+1,m}^C \cdot m}, \quad \text{and} \quad P_{t+1,m}^G = e^{-r_{t+1,m}^G \cdot m} \quad (D2)$$

The returns on the corporate and government bond are simply:

$$R_{t,t+1}^C = \ln \frac{e^{-r_{t+1,m}^C \cdot m}}{e^{-r_{t,m}^C \cdot m}} = m(r_{t,m}^C - r_{t+1,m}^C), \text{ and} \quad (\text{D3})$$

$$R_{t,t+1}^G = \ln \frac{e^{-r_{t+1,m}^G \cdot m}}{e^{-r_{t,m}^G \cdot m}} = m(r_{t,m}^G - r_{t+1,m}^G) \quad (\text{D4})$$

Rearranging the difference in return between the corporate and government bond yields:

$$R_{t,t+1}^C - R_{t,t+1}^G = -m[(r_{t+1,m}^C - r_{t+1,m}^G) - (r_{t,m}^C - r_{t,m}^G)] = -m\Delta S_{t,m} \quad (\text{D5})$$

where $\Delta S_{t,m}$ is the change in the spread from time t to $t+1$ on an m period bond. Consequently, using the

unexplained credit spread $r_{t,m}^{uc} - r_{t,m}^G$ instead of the full credit spread $r_{t,m}^C - r_{t,m}^G$ in equation (D5) and

using equation (D3), which shows that $m(r_{t,m}^{uc} - r_{t+1,m}^{uc}) = R_{t,t+1}^{uc}$ (i.e., the unexplained bond return),

equation (D5) can be rewritten as:

$$R_{t,t+1}^{uc} - R_{t,t+1}^G = -m[(r_{t+1,m}^{uc} - r_{t+1,m}^G) - (r_{t,m}^{uc} - r_{t,m}^G)] = -m\Delta S_{t,m}^{uc} \quad (\text{D6})$$

where superscript ‘‘uc’’ refers to the term ‘‘unexplained’’.

Appendix E

The After-default and After-tax Term Structures

In this section we illustrate the methodology used to compute the after-tax term structure of interest rate for corporate bonds. When assigning zero values to the tax rates, we obtain the after-default term structures that are used to compute the default spread.

If we ignore the effect of accrued interest and amortization, the price of a discount bond becomes:

$$\tilde{P}_t = \frac{\left\{ C(1-\tau_i) \sum_{m=1}^M d_{t,m} + (1-\tau_g) d_{t,M} \right\} \prod_{m=1}^M (1-\lambda_m) + \sum_{m=1}^M \left\{ \left[C(1-\tau_i) \sum_{i=1}^{m-1} d_{t,i} + (1-\tau_g) \delta d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right\}}{1-\tau_g \left[d_{t,M} \prod_{m=1}^M (1-\lambda_m) + \sum_{m=1}^M d_{t,m} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right]} \quad (\text{E1})$$

Adjusting for the accrued payments, the formula becomes:

$$\begin{aligned} \tilde{P}_t + A_t = & \left\{ (C(1-\tau_i) + A_t\tau_i)d_{t,1} + C(1-\tau_i)\sum_{m=2}^M d_{t,m} + [1-\tau_g(1-\tilde{P}_t)]d_{t,M} \right\} \prod_{m=1}^M (1-\lambda_m) \\ & + \sum_{m=1}^M \left\{ \left[(C(1-\tau_i) + A_t\tau_i)d_{t,1} + C(1-\tau_i)\sum_{i=2}^{m-1} d_{t,i} + (\delta + \tau_g(\tilde{P}_t - \delta))d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right\} \end{aligned} \quad (E2)$$

This formula applies for corporate bonds issued before July 18, 1984. After this date many modifications to the tax regulations concerning the amortization of discounts over the life of the bond require that the pricing formula be modified as is now detailed.

If the bond is held until maturity, then the amortized discount $1 - \tilde{P}_t$ is taxed as ordinary income. If the bond is sold before maturity at PS, then a number of tax scenerios are possible. First, if $PS - \tilde{P}_t < 0$, then $PS - \tilde{P}_t$ is considered a capital loss. Second, if $PS - \tilde{P}_t > 0$ and is greater than the amortized portion of the discount, then the capital gain is taxed accordingly and the accrued amortized discount is taxed as ordinary income. If $PS - \tilde{P}_t > 0$ and is less than the amortized portion of the discount, then the entire capital gain is taxed as ordinary income. Finally, in the case of default, the same logic applies except that the recovery rate δ is used instead of PS in the previous three cases.

As a result, in the case of a discount bond issued after July 18,1984, A2 becomes:

$$\begin{aligned} \tilde{P}_t + A_t = & \left\{ (C(1-\tau_i) + A_t\tau_i)d_{t,1} + C(1-\tau_i)\sum_{m=2}^M d_{t,m} + [1-\tau_i(1-\tilde{P}_t)]d_{t,M} \right\} \prod_{m=1}^M (1-\lambda_m) \\ & + \sum_{m=1}^M \left\{ \left[(C(1-\tau_i) + A_t\tau_i)d_{t,1} + C(1-\tau_i)\sum_{i=2}^{m-1} d_{t,i} + [(\delta + \tau_g(\tilde{P}_t - \delta))]d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right\} \end{aligned} \quad (E3)$$

Solving for \tilde{P}_t in (E3) yields:

$$\begin{aligned} \tilde{P}_t = & \frac{1}{1-\tau_i d_{t,M} \prod_{m=1}^M (1-\lambda_m) - \tau_g \sum_{m=1}^M d_{t,m} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i)} \\ & \times \left\{ -A_t + \left[(C(1-\tau_i) + A_t\tau_i)d_{t,1} + (1-\tau_i)d_{t,M} + C(1-\tau_i)\sum_{m=2}^M d_{t,m} \right] \prod_{m=1}^M (1-\lambda_m) \right. \\ & \left. + \sum_{m=1}^M \left[(C(1-\tau_i) + A_t\tau_i)d_{t,1} + C(1-\tau_i)\sum_{i=2}^{m-1} d_{t,i} + \delta(1-\tau_g)d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right\} \end{aligned} \quad (E4)$$

For premium bonds issued prior to September 27, 1985, the capital loss $\tilde{P}_t - 1$ can be recognized earlier using the linear amortization method. In this case, the equation for \tilde{P}_t becomes:

$$\tilde{P}_t = \frac{\left[(C(1-\tau_i) - \frac{\tau_i}{t_M-t}) \sum_{m=1}^M d_{t,m} + d_{t,M} \right] \prod_{m=1}^M (1-\lambda_m) + \sum_{m=1}^M \left[(C(1-\tau_i) - \frac{\tau_i}{t_M-t}) \sum_{i=1}^{m-1} d_{t,i} + \frac{\tau_g(t_{m-1}-t)}{t_M-t} d_{t,m} + (1-\tau_g)\delta d_{t,m} \right] \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i)}{1 - \frac{\tau_i}{t_M-t} \left\{ \sum_{m=1}^M d_{t,m} \prod_{n=1}^M (1-\lambda_n) + \sum_{m=1}^M \left[\sum_{i=1}^{m-1} d_{t,i} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right] \right\} - \tau_g \sum_{m=1}^M \left(1 - \frac{t_{m-1}-t}{t_M-t} \right) d_{t,m} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i)} \quad (\text{E5})$$

Using the constant yield amortization for bonds issued after September 27, 1985, the pricing equation becomes:

$$\tilde{P}_t = \frac{1}{1 + \tau_i \left\{ y \sum_{m=1}^M (1+y)^{m-1} d_{t,m} \prod_{n=1}^M (1-\lambda_n) + \sum_{m=1}^M \left[\sum_{i=1}^{m-1} (y(1+y)^{i-1} d_{t,i}) \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \right] \right\} - \tau_g \sum_{m=1}^M (1+y)^{m-1} d_{t,m} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i)} \times \left\{ \sum_{m=1}^M C(1 + \tau_i y \sum_{j=0}^{m-2} (1+y)^j) d_{t,m} + d_{t,M} \right\} \prod_{m=1}^M (1-\lambda_m) + \sum_{m=1}^M \left\{ \sum_{n=1}^{m-1} C(1 + \tau_i y \sum_{j=0}^{n-2} (1+y)^j) d_{t,n} + \left[(1-\tau_g)\delta - \tau_g C \sum_{j=0}^{m-2} (1+y)^j \right] d_{t,m} \right\} \lambda_m \prod_{i=1}^{m-1} (1-\lambda_i) \quad (\text{E6})$$

All the equations for the premium bonds are then modified to account for the accrued interest rate in the same way as was done for the discount bonds earlier. For further details on these adjustments, please refer to Liu et al (2005).